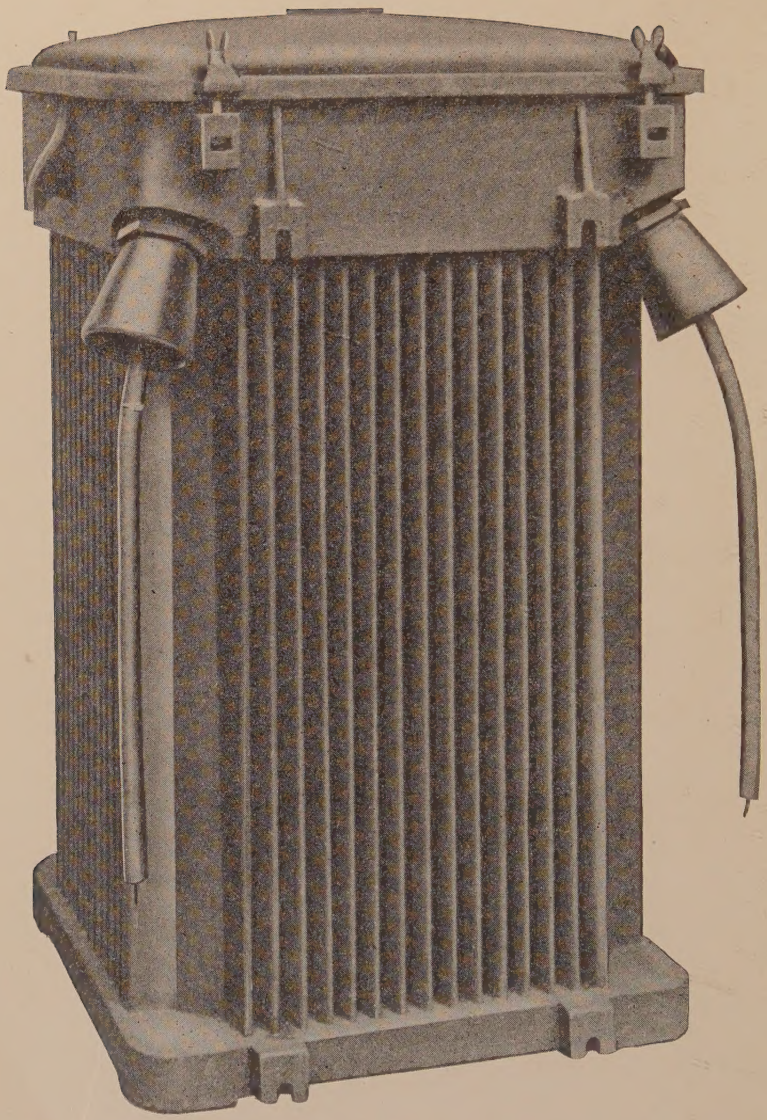

Frontispiece

THE ESSENTIALS OF TRANSFORMER PRACTICE

A TREATMENT OF THE THEORY, DESIGN AND OPERATION OF THE TRANSFORMER FROM A PRACTICAL ENGINEERING VIEWPOINT, PREPARED PRIMARILY FOR THE ENGINEERING STUDENT AND THE DESIGNING AND OPERATING ENGINEER.

By

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Illustrated

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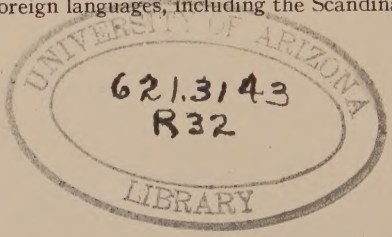

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PREFACE

A practical engineering point of view has been maintained while preparing the manuscript for this book, which discusses the theory of the transformer, the problem of design and questions of operation. Only those theories are developed, however, which have a direct bearing on design and operation. In short, the men for whom I am writing and hope to interest are engineering students and designing and operating engineers.

A series of lectures to the Transformer Section of the Electric Club of the Westinghouse Electric & Manufacturing Company was the nucleus of the following chapters. Use was also made of the matter in several papers read before the National Electric Light Association. The greater part of the material was later published in serial form in the "Electric Journal." I received a number of suggestions that the serial be published in book form, and the following is the result after revision, re-arrangement and addition of new material. Much of the text is original, although parts have been taken from such sources as have presented themselves.

I wish to acknowledge my indebtedness to the engineers of the Westinghouse Electric & Manufacturing Company with whom I have been associated, and particularly Mr. Charles Fortescue and Mr. J. B. Gibbs, for advice and suggestions which in the aggregate have been a great help in formulating the book. The editors of the "Electric Journal" were also of great assistance in getting the original manuscript ready for publication in serial form.

EMERSON G. REED.

EAST PITTSBURGH, PA.,
JAN., 1923.

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CHAPTER I

THE EVOLUTION OF THE TRANSFORMER

The evolution of the transformer began with the discovery of electromagnetic induction by Faraday and Joseph Henry. Faraday published his discoveries in 1831 and Henry announced similar and independent results during the following year. For some time each of these men was unaware of the other's investigations, one being in advance in certain discoveries, only to be anticipated by the other regarding facts equally as important. Faraday's first investigations were conducted with two solenoids, one fitting inside the other, and Henry's experiments were carried out with flat spirals. Faraday afterwards used a solid soft iron ring upon which the primary and secondary coils were wound. All of these early experiments were performed with an interrupted current from a battery, current being delivered by the secondary coil when the current was established or broken in the primary circuit.

In Faraday's ring is found the first use of the closed magnetic circuit of iron. In the subsequent development of the transformer the closed magnetic circuit was discontinued to appear later in a more perfect form. Although Faraday's ring had many of the essential features of the modern transformer, the latter cannot be thought of as having developed directly from this ring. Faraday had no conception of the transformer as a device for the economic transmission of power by stepping the voltage from a low to a higher value. As a matter of fact this ring transformer had a ratio of approximately one to one. He had no conception of the use of the alternating current, all of his experiments being carried out with an interrupted current.

The primary and secondary windings were placed on opposite sides of the iron ring, and consequently were not in close magnetic relation. When the secondary winding was carrying current there would be an enormous magnetic leakage which would prevent its successful operation as a transformer in the modern sense. Faraday used a solid iron ring which would have prevented its working at any considerable flux density, on account of the eddy currents which would have been generated. Thus we see that while this ring transformer contains many of the essential points of the modern transformer, the combination was accidental and the various elements were not put together with the direct intent of making a device for the transmission and distribution of power.

The immediate development of the transformer departed from Faraday's ring and took the form of the induction coil, the explanation of this being the fact that aside from the scientific aspect of the laws of electromagnetic induction, the object in building this apparatus was to obtain severe shocks or long sparks from the secondary winding. Professor Page of Washington in 1838 constructed an induction coil, which is essentially the induction coil of today.

In one particular the development of the transformer had retrograded in the construction of the induction coil, *i.e.*, the induction coil used an open magnetic circuit in contrast to the closed ring of Faraday. The reason for the open magnetic circuit was the simpler mechanical construction and the greater ease in insulating the secondary or high-voltage winding from the other parts of the induction coil. It is thus apparent that there were good reasons for not using the closed magnetic circuit form. At the same time there was nothing to be gained as far as the operation of the induction coil was concerned in having a closed magnetic circuit. There was no necessity for keeping a low magnetizing current, and this is the principal advantage of a closed

magnetic circuit. The closed iron circuit results in a magnetic path of low reluctance and consequently its magnetization requires only a small number of ampere-turns.

While retrograding in respect to the open magnetic circuit, the induction coil of Page contains two distinctive steps towards the modern construction. These are a divided magnetic circuit and a close magnetic relation of primary and secondary windings. By the divided magnetic circuit is meant that the core of the induction coil is constructed from a bundle of iron wires. This iron circuit permits a higher magnetic induction and hence a greater number of volts per turn than the solid iron ring of Faraday.

In 1856 Varley described an induction coil with a closed magnetic circuit consisting of a bundle of iron wires with the ends bent back on the outside of the copper coil. A construction similar to this was later used by Ferranti.

Up until about 1878 the induction coil had been used only for the production of sparks, with an interrupted current, and as yet the real function of the transformer for the distribution of electric current had not become apparent. At the Paris Exposition in 1878 Joblochkoff's system of distribution with induction coils in series was in operation with interrupted as well as alternating current. This was the last attempt at the distribution of electrical energy with the use of the interrupted current, and probably the first attempt with the use of alternating current, with the induction coil as the means of serving current to a number of points from one supply circuit. Thus a number of important steps had been introduced since the induction coil of Page. These were the return to the closed magnetic circuit by Varley, the use of the alternating current instead of the interrupted current and the conception of the induction coil as a means of serving a number of consumers of electric current from one source of supply. As yet, however, the transformer as a machine for the economic transmission of power

by means of electric current at high pressure had not been suggested.

The next step in the development of the transformer was the application of this principle, that is, the economic transmission of power. In 1882 Gaulard and Gibbs patented a system for the commercial operation of incandescent lamps from transformers, and exhibited it in London in 1883. The induction coil type of transformer connected in series was used, but later they adopted the closed magnetic circuit type of transformer shown in Fig. 1. The importance of Gaulard's and Gibbs' work lies in the early construction of a practical transformer and the application of this transformer to the economic transmission and distribution of power. The weakness of their system, and of all of the series systems used up to this time, was in its regulation. In this system the primary windings of all of the transformers are connected in series, in which case the same current flows through the primary winding of all of the transformers whether or not all of the secondaries of the transformers are loaded. If all of the transformers were always fully loaded, the regulation of the system would be satisfactory, but when the load is disconnected from one transformer, the primary current due to the load on the others must still flow in the unloaded transformer, in which case it acts as a reactance coil and absorbs a considerable amount of voltage which reduces the voltage on the remain-

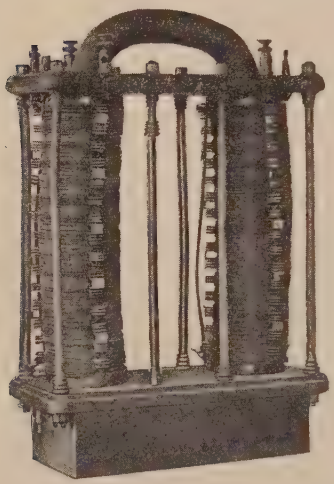


FIG. 1. Gaulard and Gibbs transformer. Original British patents dated in 1882.

ing transformers in the circuit. This was the fatal weakness of the series system and it remained for others to remove this obstacle to the successful transmission and distribution of electrical current by means of the transformer.

In 1883, Kennedy pointed out[1] that transformers operated in parallel rather than in series would be self-regulating. He did not have in mind the advantages of high potential transmission, for he considered that the size of the conductors would be an objection to the parallel system. With the primary and secondary potentials practically the same there would be no advantage over Edison's system of operating incandescent lamps in parallel with direct current. In this case the transformers would be superfluous. At this time the advantages resulting from the transmission of current at high pressure were appreciated as well as the fact that transformers when operating in parallel were self-regulating. It yet remained to combine these two concepts to form the system of alternating-current transmission and distribution which is now general.

The parallel system of constant potential high pressure distribution was brought out in this country in 1885 by William Stanley under the patronage of George Westinghouse, and in Europe during the same year by Zipernowsky, Deri and Blathy. The latter probably anticipated Stanley by a few months, but Stanley's system was immediately put into extensive practice by the Westinghouse Company. This must be considered one of the most important factors in the general introduction of the transformer. From 1883 to 1885 Stanley experimented with the converter, as the transformer was then commonly known. At that time the Westinghouse Company were considering the Gaulard and Gibbs system of series distribution. Eventually Stanley became convinced that the converter operating in parallel with

[1] In the *Electrical Review* for June 9.

high voltage primary was the proper solution of the problem, and that the Gaulard and Gibbs series system was futile. In 1885 Stanley went to Great Barrington, Mass., and after constructing a transformer of the type shown in Fig. 2, operated 150 sixteen-candle-power lamps at a distance of half a mile, the primary voltage being 500 volts.

The type of transformer built by Stanley became the commercial form used by the Westinghouse Company for many years. It was characterized by its practicability as contrasted to the types used by Zipernowsky, Deri and Blathy. The laminations of the Stanley transformer, as shown in Fig. 2, are built around the coil. The form first used by Zipernowsky consisted of the windings forming a circular ring upon which is wound the magnetic circuit in the form of soft iron wire. This form was replaced by one consisting of the magnetic circuit in circular form made of soft iron wire, upon which the windings are formed. Both of these forms were essentially impracticable, since one element must be wound upon the other by hand labor. These designs approach very closely to what may be called the "Ideal Transformer," that is, the design in which the mean turn of both iron and copper will enclose a maximum area for a given amount of material. This will result in the least amount of material and lowest cost to obtain a given performance. The ideal transformer as shown in Fig. 3 is one in which either the iron or the windings is formed into an annular ring and the other element into a circular form about this ring, completely filling its

FIG. 2.

opening. Further considerations beyond that of the materials, such as hand labor and the requirements of ventilation, space for terminals and other details of construction, prohibit the actual commercial building of the ideal transformer.

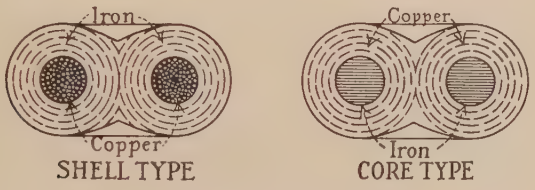


FIG. 3. Ideal transformers.

The terms "core" and "shell" type were introduced about this time to denote the difference between transformers like that of Gaulard and Gibbs, in which the iron forms a core or central portion on which the windings are placed, and those in which the iron encloses the coils like a shell, as in the Stanley form in Fig. 2. The two types of Zipernowsky, Deri and Blathy also represent the essential difference between the core and shell types of construction.

The development of the Stanley transformer in this country was rapid, although in the years of 1887 and 1888 an endeavor was made to secure legislation against the use of the alternating current for the distribution of power. The Thomson-Houston Company also brought out a shell type transformer in the year 1888, which continued to be used down to as late as 1895.

After the first commercial transformers were made, the immediate developments were improvements in detail on the shell type of construction. The use of oil as an insulating and cooling medium was generally adopted, and both the iron and copper losses were reduced and the regulation improved. The transformer was modified to meeet the requirements of 60-cycle service, the first designs being built

for 133-cycle operation, this frequency being then in general use. About 1899 to 1901 practical designs were developed indicating a close approach to the ideal transformer, as in the Berry patents, Fig. 4, taken out in England about this time.

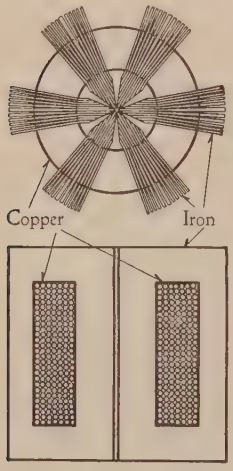


FIG. 4. Berry transformer.

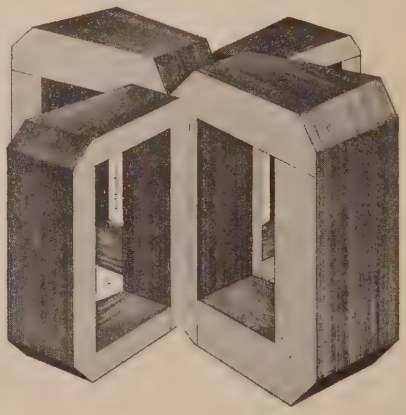


FIG. 5. Laminated iron magnetic circuit of shell-type transformer.

The modified form of the shell type for distribution transformers shown in Fig. 5 is a practical construction approaching the ideal shape. The limitations imposed by the character of the magnetic material are offset by increasing the section of the magnetic circuit outside of the winding. This permits an increase in the magnetic circuit without a corresponding change in the conductors which would have been necessary with the other form of design. A corresponding widening of the magnetic circuit outside of the windings is possible for the simple core type. This core type shown in Fig. 6 is a counterpart of the shell form in Fig. 5. In high-voltage work the core type construction, depending on the size of the transformer, finds its best field

and hence both forms of construction are justified. The efficient disposition of the insulation possible with the core type of transformer in higher voltages is the reason for its use.

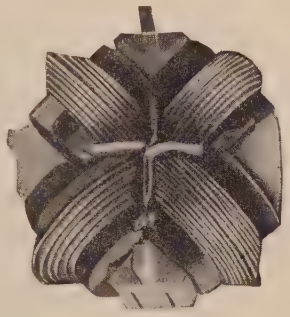


Fig. 6.

The comparatively recent advent of silicon or alloy steel has affected both the design and performance of distribution transformers. Its use does not change the relative economy of the different types of construction, but its increased cost does change the proportions of copper to iron in any particular design. The lower loss per unit of weight of the iron would naturally allow a saving of material for a given performance. This would result in an increase of the flux density in the iron and of the current density in the copper, their upper limits being set by the magnetic saturation of the iron and heating of the copper. The general effect of the better material on commercial transformers, however, has been to increase their efficiency, better their regulation and to reduce their size and weight.

CHAPTER II

COPPER LOSS

THE COPPER LOSS of a transformer is the energy loss in the copper when the transformer is loaded. It may be considered as being composed of two parts: First, the loss which is the product of the square of the current flowing in the windings and the resistance of the windings (I^2R); second, the eddy-current loss in the conductors due to the leakage magnetic flux passing through the conductors of the windings.

The I^2R copper loss is a simple matter of calculation when the resistance of the windings and the current values for which the copper loss is required are known. It is usual to specify the I^2R copper loss of a transformer at normal rated load and at some definite temperature.

CALCULATION OF THE COPPER LOSS

The current to be used in calculating the I^2R copper loss in a winding, is the watt rating of the transformer, divided by the voltage of the particular winding involved. The following relations will be found useful in calculating the resistance of transformer windings.

The resistance R of a copper conductor at 25 degrees C. may be expressed as follows:

$$R = 0.008305\frac{l}{q}. \qquad (1)$$

where l is length in thousands of feet and q sectional area in square inches.

Example 1: What is the resistance at 25 degrees C. of a copper conductor

5,320 feet long, which has a sectional area of 0.11 sq. in. From equation (1),

$$R = 0.008305 \times \frac{5.32}{0.11} = 0.4 \text{ ohms.}$$

The fundamental relation between the resistance of a copper conductor and its temperature is:

$$R_1 = R_0 \ (1 + 0.00427 t_1) \qquad (2)$$

where R_0 is the resistance of the conductor at zero C. and R_1 is its resistance at t_1 degrees C. For some other temperature R_2, equation (2) may be rewritten as follows:

$$R_2 = R_0 \ (1 + 0.00427 t_2).$$

Combining the last two equations:

$$\frac{R_1}{R_2} = \frac{234.2 + t_1}{234.2 + t_2}. \qquad (3)$$

Example 2: If a conductor has a resistance of 0.4 ohms at 25 degrees C., what is its resistance at 75 degrees C. By equation (3),

$$R_1 = \frac{234.2 + 75}{234.2 + 25} \times 0.4 = 0.477 \text{ ohms.}$$

It is often useful to think of the resistance of a transformer as being all in the high-voltage or all in the low-voltage winding. In this case, the value is called the equivalent resistance, and is defined as the quantity which, when multiplied by the square of the current in, say, the high-voltage winding of the transformer, will give the total I^2R loss. The equivalent resistance may also be the value which, when multiplied by the square of the current in the low-voltage winding, will give the total I^2R loss in the transformer. The expression for the equivalent resistance R, referred to the high-voltage winding, may be obtained in terms of the resistance of the separate windings as follows. The total I^2R loss is:

$$I_1^2 R = I_1^2 R_1 + I_2^2 R_2.$$

where I_1 and I_2 are the currents in the high- and low-voltage windings respectively, and R_1 and R_2 are the resistances of these windings. The last equation may be written:

$$R = R_1 + \left(\frac{I_2}{I_1}\right)^2 R_2 \quad \text{but} \quad \frac{I_2}{I_1} = r.$$

that is, r is the ratio of the high to the low voltage, therefore

$$R = R_1 + r^2 R_2. \tag{4}$$

Example 3: What is the equivalent resistance of a transformer, as related to the high-voltage winding, if it has a ratio of 2,200 to 220 volts or a 10 to 1 ratio, a primary resistance of 0.43 ohms and a secondary resistance of 0.0055 ohms. From equation (4),

$$R = 0.43 + 10^2 \times 0.0055 = 0.98 \text{ ohms.}$$

If this transformer has a rating of 50 k.v.a., the total I^2R copper loss will be

$$\left(\frac{50000}{2200}\right)^2 \times 0.98 = 505 \text{ watts.}$$

RELATION BETWEEN THE CURRENT DENSITY AND THE I^2R WATTS LOST PER POUND

For the purpose of computation, the I^2R copper loss may be thought of as being the product of the loss per pound of copper W_c and the total weight of copper in the transformer G_c. Then by definition:

$$W_c = \frac{I^2R}{G_c}.$$

The relation between the weight of a conductor, its length in thousand feet and its sectional area in square inches is given as follows:

$$G_c = 3{,}854\ lq. \tag{5}$$

Substituting in the equation above for W_c, the value of R from equation (1) and G_c from equation (5), we have,

since the current density in amperes per square inch is

$$I_d = \frac{I}{q},$$

$$I_d = 681.0\ W_c^{0.5} \text{ for 25 degrees C.} \qquad (6)$$

Example 4: What is the weight of a conductor which has a length of 532 feet and a sectional area of 0.11 sq. in.? From equation (5),

$$G_c = 3854 \times 0.532 \times 0.11 = 225.5 \text{ lbs.}$$

What current density in a conductor will give an I^2R loss of 4 watts per pound at 25 degrees C.? From equation (6),

$$I_d = 681.0\ (4)^{0.5} = 1362 \text{ amperes per sq. in.}$$

EDDY-CURRENT LOSS IN A CONDUCTOR OF RECTANGULAR CROSS-SECTION

The relation between the thickness of a copper conductor at right angles to the direction of a magnetic field and the resulting eddy-current loss per cubic inch, or per pound of the conductor, is developed as follows: A portion of a conductor is shown in Fig. 7, looking at it in the direction of

FIG. 7. Section of transformer conductor in the direction of the leakage flux.

the leakage flux. If its thickness i is negligible in comparison with its length, the current may be assumed to be flowing parallel with the length of the conductor, and in opposite directions on either side of the center line. It will be assumed that the eddy-currents are small enough so that their magnetizing effect may be neglected. The e.m.f. in the small path of width dx, which has unit length and depth

and which encloses an area of $2x$, is

$$\text{E.M.F.} = \frac{2}{3.5 \times 10^6} fBx.$$

where the e.m.f. is given in volts, f the frequency, in cycles per second, B the maximum field intensity in lines per square centimeter, and x in inches. This relation is based upon the law that the induced e.m.f. in a coil is equal to the rate of change of the magnetic flux through its opening.

At 75 degrees C. the resistance in ohms of the small path is given by equation (1), when corrections have been made for the temperature and the fact that the length of the circuit is expressed in inches instead of in thousand feet.

$$\text{Resistance} = \frac{0.826}{10^6} \frac{2}{dx}.$$

The current in amperes in the small path is, therefore,

$$dI = 0.346\ fBxdx$$

and the energy loss in watts due to this current is:

$$dP_e = \frac{0.198}{10^6} (fB)^2 x^2 dx.$$

or the total loss per unit length and depth of the conductor is:

$$P_e = \frac{0.198}{10^6} (fB)^2 \int_{-i/2}^{i/2} x^2 dx, \quad \text{or} \quad P_e = \frac{0.0165}{10^6} (fB)^2 i^3.$$

Since this is the loss per unit length and depth of the conductor whose thickness is i, the total loss per cubic inch is

$$\frac{P_e}{\text{cubic inch}} = \frac{0.0165}{10^6} (ifB)^2. \qquad (7)$$

or the loss per pound of conductor, since a cubic inch of

copper weighs 0.321 pound, is

$$\frac{P_e}{\text{Pound}} = \frac{0.051}{10^6}(ifB)^2. \tag{8}$$

This relation shows that the eddy-current loss per pound of the conductor is proportional to the square of the thickness of the conductor, the frequency and the density of the flux through the conductor. When conductors are formed of several wires or straps connected in parallel, the result will be approximately the same as with a solid conductor of a thickness equal to the sum of the thicknesses of the several parts. If the several parts of such a conductor are symmetrically transposed throughout their length, the voltages induced in the several parts cancel each other and the eddy-current loss is reduced to that which would exist if the parts of the conductor were in series instead of in parallel.

If the field in which a conductor is placed is not uniform, the loss in a solid conductor will no longer be the same as when it is built up of a number of wires in parallel. With a solid conductor in a non-uniform field the eddy currents cannot longer be thought of as being equivalent to a current flowing along one side of the strap and returning on the other side of the centerline. If the field intensity varies in an unsymmetrical manner, small local currents are set up which would be more or less interrupted if the conductor was laminated, even though the parts were connected together at the ends. Also in a non-uniform field, transposing the parts of the conductor will not reduce the loss to that which would exist if the conductor had a total thickness equal to that of one of the parts. The only sort of a transposition which would be entirely effective under these conditions would be to use a stranded cable, in which the various wires progress spirally through its length.

Example 5: What is the eddy-current loss per pound in a copper con-

ductor which has a thickness of 0.1 inch at right angles to a 60-cycle magnetic field of 300 lines per square centimeter? From equation (8),

$$\frac{\text{Watts}}{\text{Pound}} = \frac{0.051}{10^6}(0.1 \times 60 \times 300)^2 = 0.17.$$

EDDY-CURRENT LOSS IN TRANSFORMER WINDINGS

Since the currents in the high- and low-voltage coils of a transformer are opposite to each other in time-phase relation, at each point along the gap which separates them, there is a magnetomotive force equal to the ampere turns in either coil tending to send a magnetic flux through the gap. This flux is not confined to the gap but flows through the coils as well. The flux density is a maximum in the gap and decreases to zero at the outer edges of the coils away from the gap. In the present analysis the flux through the coils is the part of immediate interest, as this is the field which causes the eddy-current loss. As a rough approximation of the actual conditions it may be assumed that there are two main paths for this leakage flux through the windings; one returning through the magnetic circuit, as shown in Fig. 8, and one being entirely within the coil itself. Although both of these paths exist in the same coil, Fig. 8 shows for simplicity, only the path returning through the magnetic circuit in the low-voltage coil, and in the high-voltage coil only the path entirely within itself. Since the magnetic density varies from point to point in the coil, the next step is to determine the average loss per cubic inch of coil.

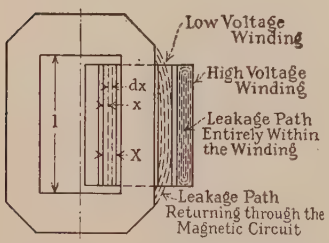


FIG. 8. Section of core-type transformer showing magnetic leakage paths.

Consider first the loss due to the leakage path which returns through the magnetic circuit of the transformer.

The magnetomotive force acting on the small width dx of the low-voltage winding shown in Fig. 8 is $(4\pi/10)\ (x/X)IT$, where x is the distance of this small section of the coil from its outside edge, X is the total radial thickness of the coil and IT its ampere turns. The reluctance of this small path to the leakage flux is $l/(l_t dx)$, where l is the length of the leakage path and l_t is the length of the mean turn of the coil. The flux through this small section of the coil is

$$d\phi = 2.54 \frac{4\pi}{10} \frac{x}{X} IT \frac{l_t}{l} dx.$$

The constant 2.54 permits the linear distances to be expressed in inches instead of centimeters. The maximum value of the flux density in the path dx is therefore,

$$B = 0.494 \sqrt{2} \frac{x}{X} \frac{IT}{l}. \tag{9}$$

Substituting this value of B in equation (7) we have

$$\frac{P_e}{\text{cubic inch}} = \frac{0.00805}{10^6} \left(if \frac{x}{X} \frac{IT}{l} \right)^2.$$

The loss in the small section of the coil whose volume is $l_t\ l\ dx$, is

$$dP_e = \frac{0.00805}{10^6} (ifIT)^2 \frac{l_t}{lX^2} x^2 dx.$$

or the loss in the entire coil in watts is

$$P_e = \frac{0.00805}{10^6} (ifIT)^2 \frac{l_t}{lX^2} \int_0^X x^2 dx,$$

or

$$\frac{P_e}{\text{cubic inch}} = \frac{0.00268}{10^6} \left(\frac{ifIT}{l} \right)^2 \tag{10}$$

or

$$\frac{P_e}{\text{Pound}} = \frac{0.00835}{10^6}\left(\frac{ifIT}{l}\right)^2. \qquad (11)$$

This expression gives the eddy-current loss in the coil per unit weight in terms of various constants of the transformer. For example, it is evident from the expression that if the frequency, thickness of the conductor at right angles to the leakage flux, or the ampere-turns in the winding be doubled (other things being equal), the eddy-current loss due to this part of the leakage flux will be four times as large as before. On the other hand if the length of the leakage path be doubled, the loss will be one-fourth of its former value.

Example 6: What is the eddy-current loss in a 60-cycle, 100 k.v.a., 2,200 to 220 volt transformer, due to the leakage path which returns through the magnetic circuit, for the following conditions?

Low-voltage, $i = 0.252$ in., 138 pounds of copper.
High-voltage coil, $i = 0.116$ in., 141 pounds of copper.

$$IT = 5220 \qquad l = 11.25 \text{ inches}$$

From equation (11),

$$\text{Low-voltage coil } P_e = \frac{0.00835}{10^6}\left(\frac{60 \times 0.252 \times 5220}{11.25}\right)^2 \times 138 = 56.5,$$

$$\text{High-voltage coil } P_e = \frac{0.00835}{10^6}\left(\frac{60 \times 0.116 \times 5220}{11.25}\right)^2 \times 141 = 12.5,$$

Total eddy-current loss $= 56.5 + 12.5 = 69$ watts.

The following analysis applies to the leakage path within the coils. For the sake of simplicity it is assumed that the path of this flux is rectangular as shown in Fig. 9.

Let

$$K = \frac{l}{X}. \qquad (12)$$

where l and X are the dimensions of the coil as shown in Fig. 9. The m.m.f. acting on this small area of width dx is

$$\text{m.m.f.} = \frac{4\pi}{10}\frac{4Kx^2}{lX}IT.$$

The reluctance of the small path is

$$\text{Reluctance} = \frac{4x}{l_t K dx} + \frac{4Kx}{l_t dx} = \frac{4x(1+K^2)}{l_t K dx}.$$

The flux through the small path is

$$d\phi = 2.54\frac{4\pi}{10}\frac{l_t}{lX}IT\frac{K^2}{(1+K^2)}xdx.$$

The maximum value of the flux density in the path dx, is therefore

$$B = 0.494\sqrt{2}\frac{x}{X}\frac{IT}{l}\frac{K^2}{(1+K^2)}. \quad (13)$$

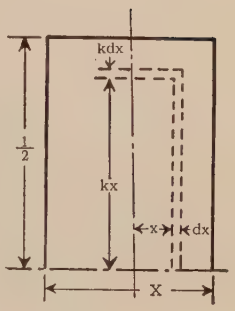


FIG. 9. Section of transformer coil at right angles to the conductors.

It is evident that this is the same expression as that in equation (9) except that the factor $K^2/(1+K^2)$ is introduced in equation (13). We may therefore, by inspection, write for the loss in the coil due to the leakage flux whose path is entirely within the coil,

$$\frac{P_e}{\text{cubic inch}} = \frac{0.00268}{10^6}\left(\frac{ifIT}{l}\right)^2\left(\frac{K^2}{1+K^2}\right)^2, \quad (14)$$

or

$$\frac{P_e}{\text{Pound}} = \frac{0.00835}{10^6}\left(\frac{ifIT}{l}\right)^2\left(\frac{K^2}{1+K^2}\right)^2. \quad (15)$$

From this expression it is evident that the eddy-current loss due to the leakage path within the coil varies the same as for the path which returns through the magnetic circuit as far as f, i, IT and l are concerned. As K increases $[K^2/(1+K^2)]^2$ grows larger and approaches unity as a limit, that is as l becomes larger and X smaller the eddy-

current loss decreases. For values of K ordinarily existing in transformers, the numerical value of $[K^2/(1+K^2)]^2$ is practically unity, which means that the loss due to the leakage flux entirely within the coil is practically equal to the loss due to the leakage flux which returns through the magnetic circuit. The total eddy-current loss in a transformer under these conditions will be approximately

$$\frac{P_e}{\text{Pound}} = \frac{0.0167}{10^6}\left(\frac{ifIT}{l}\right)^2. \tag{16}$$

Example 7: What is the total eddy-current loss in the transformer given in the previous example, when K has a value of 16?

Total eddy-current loss $= 2 \times 69 = 138$ watts.

It must not be assumed that the preceding method of calculating the eddy-current loss gives exact results in all cases, as the method outlined can be considered only as an approximation of the actual conditions when the leakage field is symmetrical. In a particular case special conditions of leakage may be introduced for which the equations will not even approximately apply.

CHAPTER III

IRON LOSS

THE IRON LOSS of a transformer is the energy required to magnetize the iron circuit. It is continuous as long as the transformer is connected to the line, and is practically independent of the load on the transformer. The loss is, however, slightly reduced under load conditions, because a small part of the impressed voltage is absorbed in overcoming the impedance of the primary winding.

When making an iron loss measurement the current taken by the transformer causes a copper loss in the winding used, which is included in the wattmeter reading. While this copper loss is not strictly a part of the iron loss, it is usually not large enough to warrant a correction for it being made. The iron loss measurement also includes a very small dielectric loss, or loss in the insulation when it is under the electrostatic stresses existing in the windings. This loss is of no practical importance except in a few cases of high-voltage transformers. The iron loss likewise depends to a slight degree on the temperature of the transformer, decreasing with increase of temperature. This is largely caused by the increase in the resistance of the iron, which reduces the eddy-currents as the temperature increases.

The apparent iron loss, as distinguished from the true iron loss, is the product of the voltage which is impressed on the winding exciting the iron and the current which flows under these conditions. The term "apparent iron loss" is self-explanatory and distinguishes the volt-ampere input, with open secondary circuit from the actual energy loss in the magnetic circuit. The ratio of the true to the apparent

iron loss, is of course the power-factor of the transformer at no load.

The iron loss will also vary with change in the shape of the voltage wave of the circuit upon which the transformer is operating. A peaked voltage wave will give a lower, and a flattened wave will give a higher iron loss than a sine wave of the same effective value. As commercial generating machinery only approximates the sine wave form of electro-motive force, the iron loss of transformers will vary when measured on circuits from different generators. For this reason it is necessary to base transformer iron loss guarantees on the sine wave.

In transformers of recent manufacture, the iron loss is a fixed quantity under given conditions, and does not increase, even though the transformer be operated with heavy loads for long periods of time. In other words, the iron in the magnetic circuit does not "age." This is due to the characteristics of the silicon steel now generally used. With the steel formerly used, there was a marked increase in iron loss from year to year. The degree of this increase depended on the temperature at which the transformer operated, and the duration of this temperature condition; the "aging" being more rapid at higher temperatures.

QUANTITATIVE EXPRESSIONS OF IRON LOSS

The curve in Fig. 10 gives the iron loss for a sample of silicon steel made into a magnetic circuit; being plotted with inductions B in lines per square centimeter of gross section as ordinates and the watts loss per pound of iron W_i as abscissae. The value of W_i for each induction may be taken as the measure of the quality of the iron in any other magnetic circuit.

Example 8: What is the total iron loss in a magnetic circuit weighing 200 pounds, which is operating at an induction of 12,000 lines per square centimeter, when excited from a 60-cycle circuit?

From the curve in Fig. 10, the iron loss in watts per pound at an induction of 12,500 lines per square centimeter is 1.7 watts. The total iron loss is therefore 1.7×200 or 340 watts. If the iron in the magnetic circuit was known to have a loss per pound of 90 percent of that shown in Fig. 10 for some particular induction, the same relation would approximately hold for all inductions and the quality of the iron in this case could be designated as 90 percent. With 90 percent iron in the example given above the total loss would be $0.9 \times 1.7 \times 200$ or 306 watts.

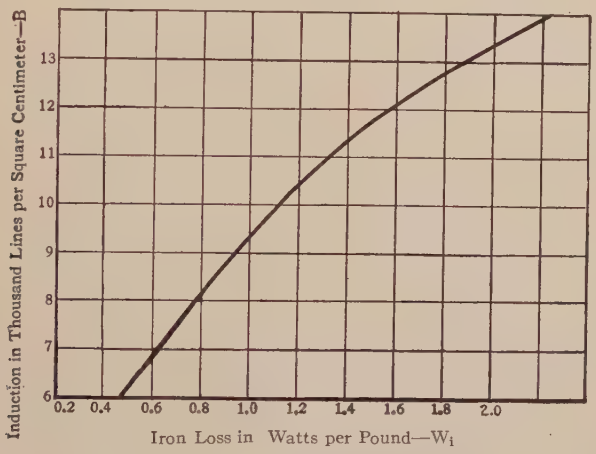


FIG. 10. Iron loss curve of silicon steel laminations at 60 cycles.

The iron loss in a magnetic circuit is made up of two parts; namely, the eddy-current and hysteresis losses. The former, like the eddy-current loss in a copper conductor, may be expressed by an equation of the form

$$\frac{P_e}{\text{Pound}} = \alpha(ifB)^2. \qquad (17)$$

where α is a constant depending on the resistivity of the steel, i is the thickness of the laminated sheets, f the frequency and B the magnetic induction in the circuit.

The hysteresis loss has been found to follow approx-

imately the empirical law.

$$\frac{P_h}{\text{Pound}} = \beta f B^{1.6}. \tag{18}$$

where β is a constant depending on the grade of the steel. The total iron loss in the circuit may therefore be written

$$\frac{P_i}{\text{Pound}} = W_i = \alpha(ifB)^2 + \beta f B^{1.6}. \tag{19}$$

This expression is rather complicated and difficult to handle in developing relations showing the variation of the iron loss of transformers with voltage, frequency and other similar quantities. For this purpose the following exponential function is used which expresses, with fair accuracy, the iron loss per pound of laminated transformer iron, within rather narrow limits of induction. Where the value of the induction is considerably changed a different value of the exponent must be used.

$$W_i = a f^x B^y \tag{20}$$

Where a is a constant depending on the quality of the steel and x and y are variable exponents considering a wide range of the frequency and magnetic induction, but constants for a limited range of these quantities.

The use of this simple function to represent the iron loss, permits many useful and interesting relations pertaining to the transformer to be developed. Where greater accuracy is required a more complicated function must be used, which naturally limits more or less its general application.

The value of y is determined for any given sample of iron by plotting the values of the iron loss W_i and B for that particular sample, to logarithms. The value of y is then the slope of this curve, as is evident from the following. For any particular value of the frequency f equation (20)

may be written,

$$W_i = \text{Constant } B^y$$

whence

$$\log W_i = \log \text{Constant} + y \log B,$$

and for another density B'

$$\log W_i' = \log \text{Constant} + y \log B'.$$

Combining these equations,

$$y = \frac{\log W_i - \log W_i'}{\log B - \log B'}. \qquad (21)$$

Example 9: What is the value of the exponent y of the iron loss curve shown in Fig. 10, at an induction of 9,500 lines per square centimeter? The curve in Fig. 10 may be plotted to logarithms, whose slope between the limits of inductions of 9,000 and 10,000 is approximately as follows from equation (21).

$$y = \frac{0.050 + 0.0343}{4.000 - 3.954} = 1.83.$$

In Fig. 11 the value of this exponent is shown for each point of the iron-loss curve shown in Fig. 10. The values of y corresponding to the upper end of the iron-loss curve

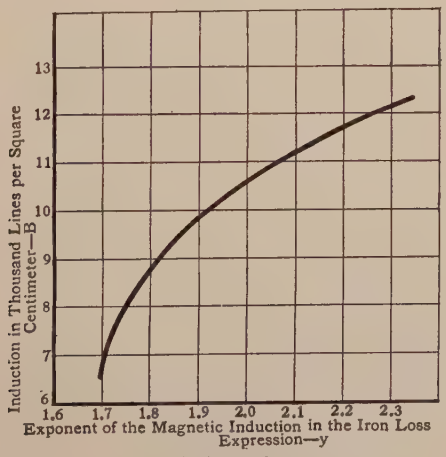


FIG. 11. Variation of exponent y.

are somewhat indefinite, as slight differences in the value of the W_i used from this region make a considerable difference in the value of y as determined from equation 21.

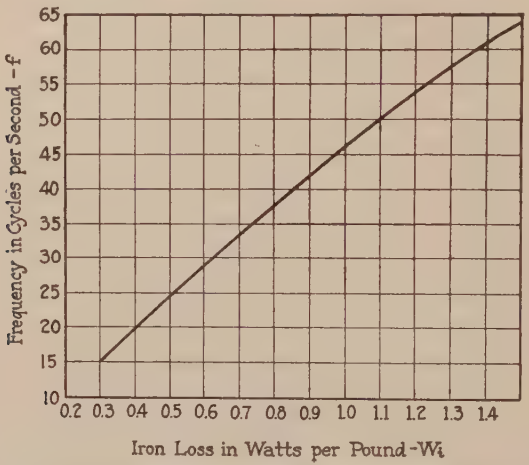


FIG. 12. Iron loss of the same sample of silicon steel as Fig. 10. Magnetic induction constant at 11,000 lines per square centimeter.

Fig. 12 shows a curve which gives the relation between the watts per pound and the frequency, with a constant magnetic induction of 11,000 lines per square centimeter. This curve can be used to determine the frequency exponent x, in the same manner as the curve in Fig. 10 is used to determine y, the exponent of the magnetic induction. Fig. 13 shows the variation in x for a large range in the value of the frequency.

In specifying the magnetic characteristics of transformer iron, it is necessary to know the apparent watts which will be required to maintain a certain flux density in the iron, as well as the actual energy loss which will result. The simplest method to accomplish this is to determine a curve similar to that shown in Fig. 10, but instead of plotting true iron loss and induction, apparent loss and induction

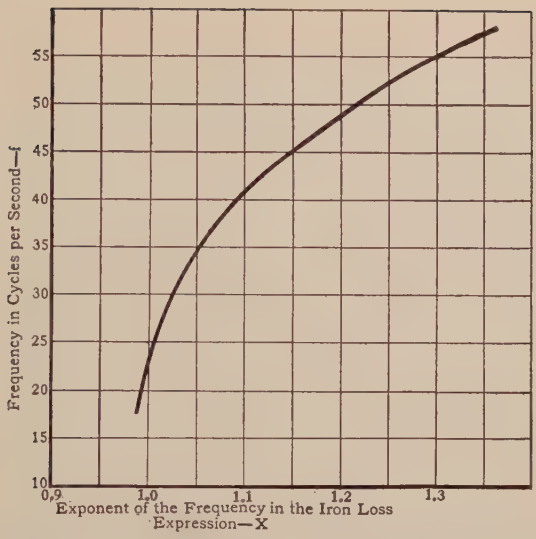


FIG. 13. Variation of the exponent x, showing that the exponent of the curve in Fig. 12 is not constant for large variations of the frequency.

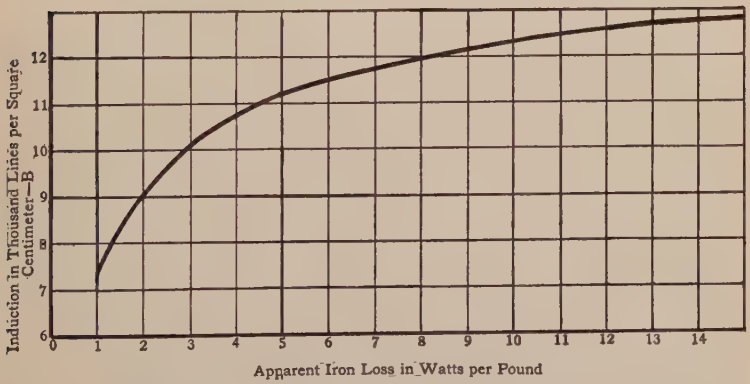


FIG. 14. Apparent iron loss curve for a sample of silicon steel at 60 cycles.

are used. Such a curve for silicon steel is shown in Fig. 14, which is used in exactly the same way as the iron-loss curve shown in Fig. 10. Since the iron-loss curve in Fig. 10 and the apparent iron-loss curve in Fig. 14 are for the same

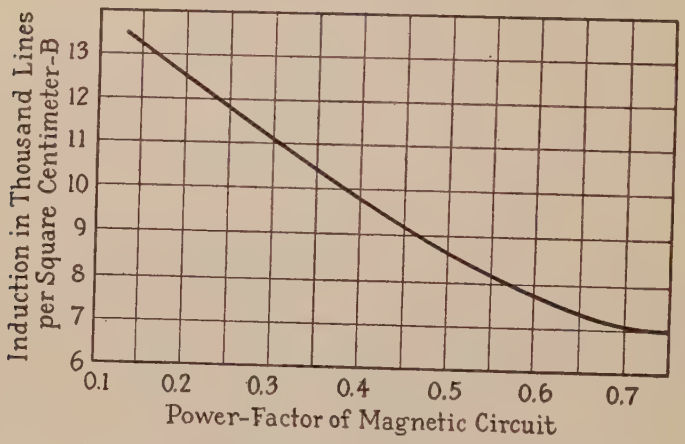


FIG. 15. Variation of power factor and induction.

sample of material, the ratio of the two loss values at the same induction gives the power-factor at that point. The curve in Fig. 15 shows the variation of the power-factor of the magnetic circuit with the induction.

SEPARATION OF THE IRON LOSS INTO ITS EDDY CURRENT AND HYSTERESIS COMPONENTS

The study of the eddy-current and hysteresis losses separately is important as a research problem in investigating the characteristics of different grades of steel and in a practical way with the transformer to determine when the insulation between the laminations is effective.

In developing the expression for the eddy-current loss it is assumed that the eddy currents flow parallel to the surface of the plates and that the impedance of the eddy-cur-

rent circuits is made up of resistance only. Actually the eddy currents do not always flow parallel to the surface of the plates, especially near where the ends of the laminations butt against each other. At these points the flux fringes out and flows through the sides of the adjacent plates and sets up local eddy currents. Also when the frequency is high the reactance of the eddy-current paths will not be negligible and the eddy-current loss will then vary less than as the square of the frequency, when the flux density is kept constant. If the frequency be carried high enough to make the impedance practically independent of the resistance, the eddy-current loss will become independent of the frequency.

If the resistivity of silicon steel be taken as $60/10^6$ ohms per cubic centimeter, or $23.6/10^6$ ohms per cubic inch, the eddy-current loss per pound by the same method used in developing the equation for the eddy-current loss in a conductor,

$$\frac{P_e}{\text{Pound}} = \frac{0.0024}{10^6}(ifB)^2 \qquad (22)$$

or

$$\alpha = \frac{0.0024}{10^6}.$$

This is based on the silicon steel having a specific gravity of 7.5 and an allowance for spacing between plates of approximately 15 percent. It is interesting to note the relation between the eddy-current loss per pound in a copper conductor and in a silicon-steel plate under the same conditions of thickness of sheet frequency and induction. This is equal to the ratio of the eddy-current coefficients for the two materials, or

$$\frac{\text{Watts per pound of copper}}{\text{Watts per pound of steel}} = \frac{\frac{0.051}{10^6}}{\frac{0.0024}{10^6}} = 21.4.$$

Example 10: What is the eddy-current loss in a magnetic circuit composed of 400 pounds of steel operated at 60 cycles and at 10,000 lines per square centimeter; the thickness of the laminations being 0.014 inch? From equation (22),

$$P_e = \frac{0.0024}{10^6}(0.014 \times 60 \times 10000)^2 \times 400 = 67.7 \text{ watts}$$

The above method for approximating the eddy-current loss is based on first priciples, the only experimental data required being the resistivity of the material. The bysteresis loss can not be calculated in this manner and a predetermination of its value in a particular case must be based upon the results of previous tests. The iron loss of a particular magnetic circuit may be separated into its eddy-current and hysteresis components, but more than one measurement is necessary. The following is one of the several methods of separating the losses, by measurements taken at two different frequencies. Let W_i and W_i' be two measurements of the iron loss per pound, made at two different frequencies f and f_1 and at the same flux density. Equation (19) may then be written for each of these measurements.

$$W_i = \alpha\ (ifB)^2 + \beta f B^{1.6},$$
$$W_i' = \alpha\ (if_1B)^2 + \beta f_1 B^{1.6}.$$

Combining these equations and solving for α, we have

$$\alpha = \frac{W_i'f - W_if_1}{i^2B^2(ff_1^2 - f_1f^2)}. \qquad (23)$$

Example 11: If a transformer has an iron loss of 0.432 watts per pound at 25 cycles and 1.122 watts per pound at 60 cycles and at an impressed voltage in each case such as to give an induction of 10,000 lines per square centimeter in the magnetic circuit made up of laminations 0.014 inch thick; what part of the loss at 60 cycles is due to eddy currents and what part is due to hysteresis loss? From equation (23),

$$\alpha = \frac{1.122 \times 25 - 0.432 \times 60}{(0.014 \times 10000)^2(25 \times 60^2 - 60 \times 25^2)} = \frac{0.00214}{10^6}$$

which is in reasonably close agreement with the coefficient developed from

the resistivity of the steel. The eddy-current loss then, from equation (6), is

$$\frac{P_e}{\text{Pound}} = \frac{0.00214}{10^6}(0.014 \times 60 \times 10000)^2 = 0.152.$$

In this particular case the eddy-current loss is $(0.152/1.122) \times 100$ or 13.7 percent of the loss at 60 cycles. The hysteresis loss is therefore $1.122 - 0.152$ or 0.97 watt per pound. The hysteresis coefficient is therefore from equation (18),

$$\beta = \frac{0.97}{60 \times 10000^{1.6}} = \frac{0.00645}{10^6}.$$

The hysteresis loss for any other induction may now be calculated, assuming that it varies as the 1.6 power of the induction. As a matter of fact the exponent 1.6 holds only for medium values of the induction. At very low densities the exponent may increase to *2* or more and at high flux densities to as high as *3*. The reason for the increased exponent at higher inductions is partly that the scale on the sheets carries an appreciable part of the flux at these densities, the higher loss coefficient of the scale then producing a considerable increase in the total hysteresis loss.

THICKNESS OF THE LAMINATED STEEL PLATES TO MAKE THE TOTAL IRON LOSS A MINIMUM

If the total volume of the magnetic circuit is constant and there is a fixed insulation distance between plates, as the thickness of the laminations decreases the actual magnetic density will be increased. Thus the hysteresis loss will increase, but because of the reduction in the thickness of the sheets, the eddy-current loss will be reduced. It is therefore evident that for some thickness of the sheets, the total iron loss will be a minimum. If equation (19) represents the actual conditions with sufficient accuracy, it is possible to determine this thickness theoretically. A determination of this sort give thicknesses smaller than those ordinarily used. Actual tests however indicate considerable variation in the value of the constants in equation (19) with different

conditions of frequency, flux density and thickness of laminations. The selection of the proper thickness of sheet becomes therefore more a matter of test than of theory, particularly since the cost of rolling and handling the material varies with the thickness of the plate. The thickness ordinarily used in transformers varies from approximately 0.0125 to 0.0179 inch. Other things being equal, it is evident that the lower the eddy-current loss as compared to the hysteresis component, the thicker is the sheet which can be economically used.

EFFECT OF THE FORM FACTOR OF THE VOLTAGE WAVE ON THE IRON LOSS AND APPARENT IRON LOSS

The induced voltage wave in a transformer winding has of course the same shape as the impressed voltage. The induced voltage at any instant is the rate of change of the flux in the magnetic circuit, which is another way of saying that the flux wave is the time integral of the voltage wave. Imagine the voltage wave to be distorted by the presence of a third harmonic which gives a peaked wave shape. It is evident that the time integral of such a voltage wave will give a flattened flux wave, or a flattened voltage wave will give a peaked flux wave. The reason is then apparent why a flattened voltage wave will give an increased iron loss and a peaked voltage wave a reduced loss in the transformer. Further since the flux wave is the time integral of the voltage wave, the maximum value of the flux or the induction in the magnetic circuit is proportional to the mean value of the voltage wave. Also since the form factor of the voltage wave is the ratio of its effective value to its average value, if its effective is maintained constant, the average value of the voltage wave is inversely proportional to its form factor. In other words with the effective value of the voltage constant,

$$B_{\max.} \propto E_{\text{av.}} \propto \frac{1}{\text{form factor}},$$

therefore,

$$\frac{P_h}{\text{Pound}} = \beta f \left(B \frac{1.11}{\text{form factor}} \right)^{1.6} \quad (24)$$

A peaked voltage wave gives an increased form factor and a flattened wave a reduced form factor. The curve in Fig. 16 shows the variation of the hysteresis loss with the

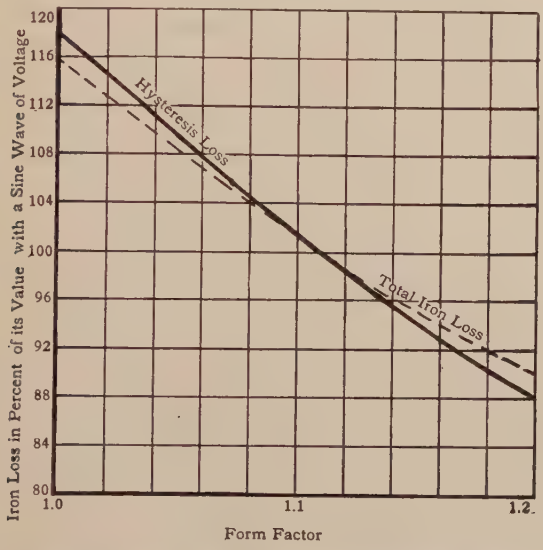


FIG. 16. Effect of form factor of the voltage wave on the hysteresis and total iron losses of silicon steel, based on 15 percent of the total loss being eddy-current loss.

range of form factor likely to be met with in practice. Ordinarily the wave form is peaked but the average variation would not be as much as ten percent. Assuming the average voltage wave to have a form factor three percent greater than that of a sine wave, the curve in Fig. 16 indicates that the hysteresis loss will be about 95.7 percent of the value as measured on a sine wave of voltage. Since the eddy-current is a function of the effective voltage in-

dependent of the wave form, the change in iron loss due to a change in the form factor of the voltage wave, is due entirely to the change in the hysteresis loss. Therefore the change in the total iron loss, as shown by the curve in Fig. 16, is less with a given variation of the form factor, than the change in the hysteresis loss. With an increase of three percent in the form factor above that of the sine wave the total loss is approximately 96.2 percent of that on a sine wave. The curve in Fig. 16 is based on the induction in the magnetic circuit being such that the hysteresis exponent is 1.6. At higher inductions the exponent being larger, the change in the iron loss is more rapid with change in the form factor of the voltage wave.

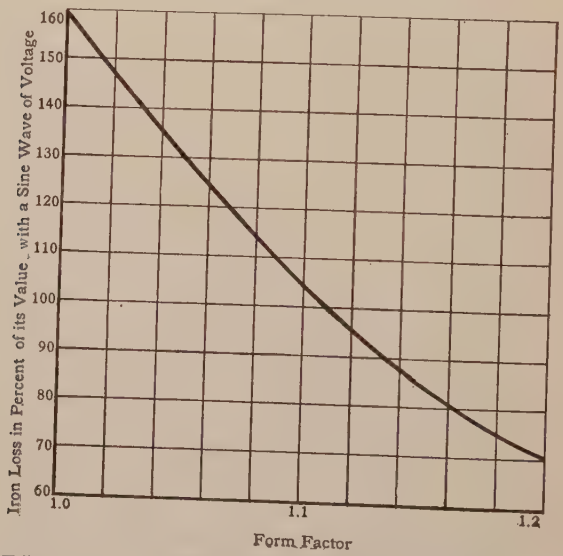


FIG. 17. Effect of form factor of the voltage wave on the apparent iron loss of silicon steel.

Since the apparent iron loss in a transformer is a function of the maximum value of the induction, the effect of a

change in the form factor can be determined from the apparent iron loss curve in Fig. 14. Fig. 17 shows the variation of the apparent iron loss with the form factor, assuming that with a form factor of 1.11 the induction is 10,000 lines per square centimeter. If the average commerical form factor is three percent higher than that of a sine wave, the apparent iron loss of a transformer will be 87.5 percent of the loss on a sine wave of voltage.

The determination of the ratio (1.11/form factor) in equation (24) for a particular case, is a laboratory rather than a commercial measurement. It is equal to the ratio of the average value of the rectified alternating distorted voltage, to the corresponding average of the sine wave voltage. The most practicable commercial method of getting an iron loss measurement on a sine wave basis is to measure the iron loss of a transformer on a true sine wave, and then find the effective voltage on the distorted wave which will give the same loss measurement. This voltage may then be used to measure the iron loss of other transformers built with steel which has approximately the same ratio between the eddy-current and hysteresis losses, as in the auxiliary transformer. This is the principle of the iron loss voltmeter.

OPEN-HEARTH AND SILICON STEELS[1]

The characteristics of the open-hearth and silicon steels are as follows:

	Open-hearth steel	Silicon steel
Percentage of silicon	0.01 to 1.0	2.5 to 5.0
Specific gravity	7.7	7.5
Eddy current in percent of total loss at 10,000 gausses	30.0	15.0
Hysteresis loss in percent of total loss at 10,000 gausses	70.0	85.0

[1] See article by Mr. T. Spooner in *The Electric Journal* for March, '17, p. 90.

	Open-hearth steel	Silicon steel
Total loss in percent of the loss in open-hearth steel at 10,000 gausses	100.0	55.0
Maximum permeability	5,000 to 6,000	6,000 to 7,000
Permeability at 16,000 gausses	500	250
Power-factor at 10,000 gausses	0.64	0.40
Permanence of characteristics	Ages or deteriorates with time.	Non-aging or does not deteriorate with time.

The silicon content imparts the non-aging quality and this is not the least important property of this steel. The silicon also reduces the hysteresis as well as the eddy-current loss. The eddy-current loss is reduced because of the increase in the resistance of the steel by the silicon content. The objections to the silicon steel are that rolling and punching are made more difficult and its permeability is lower at high densities. At medium densities, however, its permeability is comparable to that of the open-hearth steel. At an induction of 10,000 lines the power-factor of the silicon steel is only approximately 60 percent of that of the other grade; this is because the true loss element of the silicon steel is lower and not because the apparent loss element is greater. As a matter of fact the apparent loss curves for the two grades of steel are quite close together for medium values of the induction.

VARIATIONS OF IRON LOSS AND APPARENT IRON LOSS IN COMMERCIAL TRANSFORMERS

The apparent iron loss of transformers is much more subject to variation with different units than the true iron loss. While from five to ten percent variation might be usual with regard to the true iron loss, with the same grade of steel in the magnetic circuit, twenty-five percent or more would more likely represent the normal variation of the apparent iron loss. Slight differences in the amount of

steel used and in the closeness of the gaps in the magnetic circuit have a greater effect on the apparent than on the true iron loss. When the losses of a transformer on test do not come within these limits, the trouble can usually be traced to a poor intrinsic quality of the steel used, shortage of steel, poor building, laminations not properly insulated from each other, circulating currents in the windings, mixture of two or more grades of steel of different permeability, or cross flux.

The intrinsic quality of the steel depends on its chemical composition and on its subsequent heat and mechanical treatment. The heat treatment or annealing cycle owes its beneficial effect largely to the removal of the stresses existing in the sheets. After the sheets come from the rolls, an annealing process considerably increases the permeability and reduces the hysteresis loss. After the material has been punched and made ready for building into the transformer it is given another annealing cycle to remove the stresses resulting from this operation. When the very best results are desired a further annealing operation is sometimes advisable. Since the eddy-current loss depends only on the resistivity of the material it is not effected by mechanical or heat treatment. Mechanical stresses exerted externally on the punchings have an effect similar to the strains existing in the sheets from rolling and punching. The direction of the grain in the steel from rolling has also an appreciable effect on the permeability and hysteresis loss in relation to the direction of the flux through the sheet. The best results are secured when the direction of the flux is in the direction of the grain. If an L punching is used the best condition is to have the direction of the grain at an angle of 45 degrees to the direction of the flux. The direction of the grain in relation to that of the flux has no effect on the eddy-current loss.

When gaps are introduced into a magnetic circuit by poor

building, its magnetization curve, in addition to the characteristics of the material, begins to partake of the magnetization characteristics of an air gap; that is, the curve is flattened out and does not have so decided a knee. The iron loss will also be greater because of the slight fringing of the flux at the gaps, which increases the eddy-current loss in the sheets adjacent to the joint. It is possible to place the joints in a magnetic circuit, so that the characteristics of the circuit will not be greatly different from a circuit without gaps. The number of gaps should of course be as small as possible, not more than two in a single-phase transformer, and they should be placed if possible in the low density part of the circuit. A shortage of steel in the magnetic circuit will increase the iron loss by about the same percentage as the circuit is short in weight. The apparent iron loss will be increased at a very much higher rate than this, particularly at high inductions. The increased loss is due to the loss per pound increasing at a faster rate, due to the increased induction, than the total loss is reduced by the smaller number of pounds of steel.

For the laminations of small transformers where the pressure of one plate against the other is not great, the oxide on the laminations, together with the compound which penetrates between them during impregnation, is sufficient to prevent appreciable loss due to eddy currents flowing from plate to plate. For the larger sizes of transformers, where the area of the laminations in contact with each other is relatively large, and there is some pressure, it is found advisable to japan the separate punchings.

Circulating currents may appear in the windings as a result of paralleling different windings, which have slightly different voltages. The unbalanced voltages causing these currents and consequently the currents themselves will increase directly as the induction in the magnetic circuit, and the copper loss due to the currents will increase as the square of the induction.

If the magnetic circuit is made up of two kinds of iron, which have different permeability characteristics, the flux will seek the iron of high permeability and this will probably increase the total loss, particularly if the iron of higher permeability has a higher loss per pound.

If the magnetic circuit is composed of two or more parts in parallel which do not have the same relative areas at all points, cross-fluxes will be set up where the relative areas change, and as the flux crosses through some of the laminations, eddy currents are produced which increase the iron loss. The eddy currents have a tendency to damp out this flux thus preventing an extremely large increase in the loss.

Increased loss due to the quality of the iron, to gaps between the ends of the laminations, to eddy currents between plates, and to circulating currents in the windings will give an iron-loss curve for the transformer which will be a multiple of a sample of the iron which makes up its magnetic circuit. On the other hand, if an iron-loss curve for the transformer departs markedly at some voltage from that of a sample of the iron, it is reasonable to assume that there is a cross-flux in the circuit due to one of the causes referred to.

TRANSFORMER EXCITING CURRENT

The exciting current of a transformer is defined as the ratio of the apparent iron loss taken at normal voltage to the normal rated output of the transformer. The value of the exciting current of a transformer is an important element of its electrical performance as will be shown later. The curve shown in Fig. 18 is a typical magnetization curve of a transformer. This curve is usually drawn between impressed voltage as ordinates, and exciting current in amperes as abscissae. The value of the magnetization curve is that it shows the voltage at which the magnetic circuit of the transformer approaches saturation.

The shape of the exciting current wave of a transformer has an important bearing on its performance under certain conditions of operation, as will be shown later. This curve does not have a sine-wave form, even when the transformer is excited with a sine-wave of e.m.f. This distortion is due to the varying permeability of the magnetic circuit at different inductions. The sine-wave form of e.m.f. necessitates a sine-wave form of magnetic flux. Since the permeability of the iron varies with the different value of the flux, or the flux density when a particular magnetic circuit is involved, the current required to establish this flux also varies. In order to develop the actual exciting current in a particular case it is necessary to have a B-H curve for the magnetic circuit for both ascending and descending magnetizations,

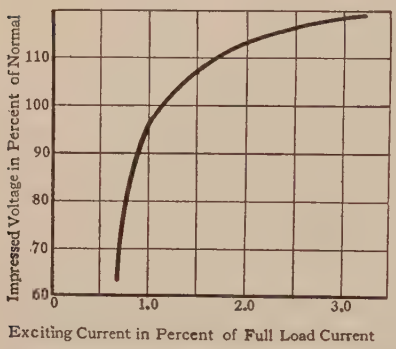


FIG. 18. Magnetization curve of a 25 k.v.a., 2,300 to 230–115 volt, 60 cycle transformer.

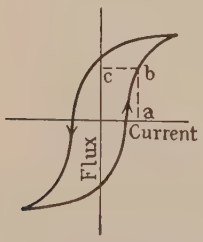


FIG. 19. Hysteresis loop.

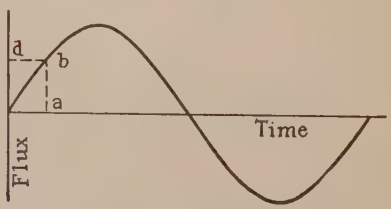


FIG. 20. Harmonic flux curve.

or in other words a hysteresis loop. In Fig. 19 let the ordinates of the hysteresis loop represent values of the magnetic flux produced by various current strengths in the primary winding of the transformer. In Fig. 20 is shown

the harmonic flux curve resulting from the harmonic e.m.f. impressed on the transformer. In this figure ordinates represent flux, and abscissae represents time or angular rotation. Fig. 19 thus gives the relation between flux and current, and Fig. 20 the relation between flux and time, and it is desired to establish the relation between current and time. For a value of the flux *a-b* in Fig. 19, the corresponding value of the current is *c-b*, and for the same value of the flux in Fig. 20, the value of time is *b-d*. Now plotting these values of time and current in Fig. 21 gives the point *b*, the locus of which is the required curve. The ascending branch of the hysteresis loop is used for increasing values of the flux and the descending branch for decreasing values. It is evident that the exciting current wave could be made a sine wave by impressing a distorted voltage wave, which contains suitable harmonic components. This is important, as there are operating conditions which will not permit the complex exciting current wave being supplied to the transformer from the line.

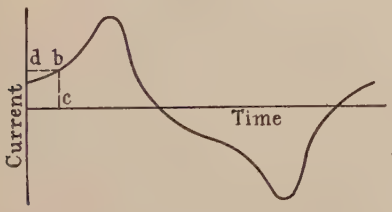


FIG. 21. Exciting current curve, combining Figs. 19 and 20.

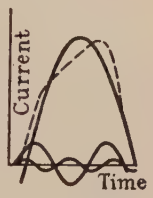


FIG. 22. Complex current curve.

For the purpose of analysis, this complex current wave may be resolved into its component curves, of the first, third and fifth frequencies, as shown in Fig. 22. The amplitude of the third harmonic increases with the magnetic flux density; the limiting value being dependent to a certain extent on the value of the other higher harmonics also present

in the current wave. In general, the higher the magnetic flux density the more prominent are the higher harmonics. Since the two half waves in a cycle of the complex current wave are identical except as to sign, the wave cannot contain even harmonics, as the odd harmonics alone can produce this condition.

CHAPTER IV

GENERAL RELATIONS

THE LOSSES in a transformer can be varied by changing the voltage impressed on its primary winding, the load current at the same time being adjusted so that the k.v.a. output remains constant. If the secondary voltage is decreased, the induction in the magnetic circuit, with the impressed voltage, decreases and decreases the iron loss. When the voltage is low, the current is large and consequently the copper loss is increased and when the voltage is high, the current and copper loss are low. Practically there is a limit to the extent to which the voltage may be increased due to saturation of the magnetic circuit, and a limit to the increase in current output due to heating of the windings.

RELATION BETWEEN THE COPPER AND IRON LOSSES, THE IMPRESSED VOLTAGE BEING VARIED

For a transformer having a given k.v.a. output at a primary voltage E, the primary load current I corresponding to this voltage may be expressed thus:

$$I = \frac{\text{k. v. a.}}{E}. \tag{25}$$

Since the copper loss of a transformer, not counting the eddy-current loss, is proportional to the square of the current in its windings, its value L_c will then be

$$L_c \propto \frac{I}{E^2}. \tag{26}$$

The copper loss curve in Fig. 23 shows in graphical form the relation expressed in equation (26), the impressed

voltage and copper loss being given in terms of their normal value; or the figures for which the transformer was designed. Equation (20) shows that at a given frequency the iron loss in a transformer is proportional to B^y and since the induction B in the magnetic circuit is proportional to the voltage E impressed on the winding, this relation may be written:

$$L_i \propto E^y. \tag{27}$$

The iron-loss curve in Fig. 23 shows the relation expressed by equation (27), the impressed voltage and the iron loss being given in percent of their normal values and for $y = 2$.

From the curves in Fig. 23 it is apparent that, as the iron

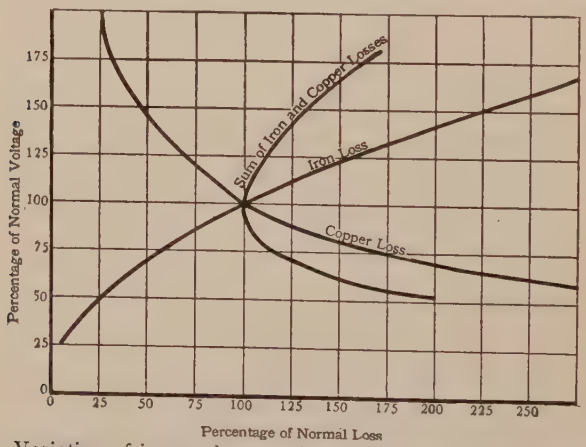


FIG. 23. Variation of iron and copper losses of a transformer, output being constant and the impressed voltage variable.

loss becomes small, the copper loss increases very rapidly, and when in turn the copper loss reaches low values the iron is increasing rapidly. The sum of the percentage iron and copper losses is also shown in Fig. 23 and from the three curves it may be seen that as the percentage iron loss in-

creases and the percentage copper loss decreases, the sum also decreases until a minimum is reached, after which it again increases. The condition for the sum of the percentage losses to be a minimum has no relation to the requirement for the sum of the losses in watts to be a minimum. The voltage which will make the sum of the losses in watts a minimum depends on the ratio of the iron and copper losses at normal voltage.

The relation between the iron and copper losses is secured by combining equations 26 and 27 which gives

$$L_c^{1/2} L_i^{1/y} = \text{Constant.} \qquad (28)$$

A further discussion of this relation is given under "total equivalent loss."

RELATION BETWEEN THE COPPER AND IRON LOSSES, THE TURNS IN THE WINDINGS BEING VARIED

Instead of changing the losses of a transformer by impressing a variable voltage on its primary winding and keeping its output constant, the same result may be accomplished with a constant impressed voltage, by varying the number of turns in its windings, keeping the volume of the copper constant. It is assumed that the frequency at which the transformer is operating is constant in both cases. The relation between the number of turns T in the coil which excites the magnetic circuit and the flux density B in lines per square centimeter is given by the following equation, which may be characterized as the fundamental equation of the transformer,

$$B = \frac{3.5 \times 10^6}{f} \times \frac{E}{TA_i}, \qquad (29)$$

where A_i is the area of the magnetic circuit at right angles to the flux, expressed in square inches, f is frequency of the current, and E is the voltage impressed on the winding.

This relation is based on the law that the induced voltage in each turn of a coil is equal to the rate of change of the magnetic flux through its opening. Equation (29) indicates than an increase in the number of turns T will decrease the induction B in the magnetic circuit and consequently the iron loss. Decreasing T, on the other hand, will increase the iron loss, the impressed voltage being constant. With a constant total volume of winding, an increase in the number of turns of the conductor necessitates a reduction of its sectional area and an increase of its length. If the number of turns is decreased, a shorter length of conductor having a greater section results. As the resistance of a conductor increases as its length becomes greater and its cross-section less, the copper loss increases with the number of turns in the winding; and conversely a reduction in the number of turns decreases the copper loss.

Ignoring the eddy-current copper loss, the I^2R loss is proportional to the equivalent resistance R, since the current output I remains constant. That is,

$$L_c \propto R \propto \frac{l}{q},$$

where l is the length of the conductor in the winding, q is its sectional area.

As the number of turns in the winding varies, it is assumed that the space factor of the coil is constant, the space factor being defined as the percentage of the total sectional area available for the winding actually occupied by conductors; or in other words, as the sectional area of the conductors multiplied by the total number of turns and divided by the opening in the magnetic circuit at right angles to the conductors. As the number of turns in the winding increases, the insulation on the conductors becomes a somewhat greater percentage of the total available winding section and thus the space factor is not constant as the number

of turns varies. The change in the space factor is not great for a limited variation in the number of turns and may be considered constant without introducing any considerable error.

Since the space factor remains approximately constant, the total volume of the copper does not change and the sectional area of the conductor necessarily varies inversely as its length. Therefore the above equation may be written:

$$L_c \propto R \propto \frac{l}{\frac{1}{l}} \propto l^2.$$

Since the mean turn of the winding does not change, the number of turns T is proportional to the length l of the conductor or

$$L_c \propto T^2.$$

Since T is inversely proportional to B from equation (29), this may be written:

$$L_c \propto \frac{1}{B^2}. \qquad (30)$$

This is the same relation as expressed by equation (26), since E in that expression is proportional to B, when B is changed by varying E, the impressed voltage. As the iron loss is a function only of the induction, the iron and copper losses must have the same relation to each other in both cases.

It may therefore be stated, as a general proposition, that the relation of the losses is the same when they are varied by changing the impressed voltage, keeping the output constant, as when they are varied by changing the number of turns, keeping the impressed voltage and the volume of the copper constant. Of course the frequency of the supply current is assumed to be constant in both cases.

EQUIVALENT TOTAL LOSS

The general relation shown in equation (28) is very interesting and is one from which further useful conclusions may be drawn. When y equals 2 equation (28) becomes

$$L_c L_i = \text{constant}, \qquad (31)$$

which may be interpreted as follows: A transformer having a magnetic circuit and winding of fixed dimensions, with a constant rating at a definite frequency, has losses such that the product of the copper and iron losses is a constant quantity, the change in the relative values of the iron and copper losses being produced by varying the voltage impressed on the transformer. When the output remains constant this results in a varying current in the winding and the

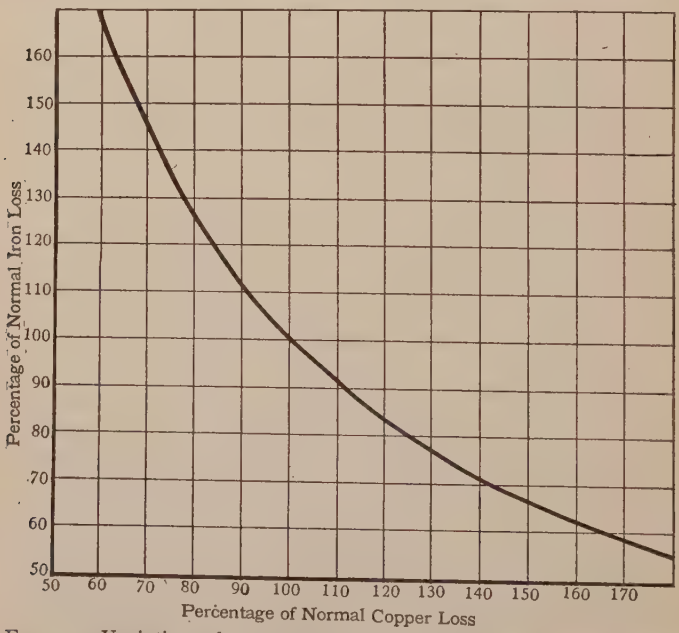


FIG. 24. Variation of percentage of normal iron and copper losses for a given transformer when the iron loss is varied by changing the impressed voltage. The output of the transformer is constant.

magnetic circuit is operated at different inductions and the copper at various current densities. It has been shown that the variations of the losses follow this same law when the number of turns in the winding is changed, keeping the impressed voltage and current output as well as the volume of the copper constant.

When the normal iron and copper losses of a transformer are known, and some condition is assumed which gives a new value for one of the losses, the unknown iron or copper loss may be calculated from this relation. This relation is a general one and may be plotted as shown in Fig. 24, which gives the percentage of normal iron loss resulting from various values of percentage of normal copper loss.

Since the product of the losses for a given transformer is constant as their relative value changes, this value gives a true basis of comparison for the relative merits of two transformers as far as their iron and copper losses are concerned. This is the reason for the product of the losses being designated as the equivalent total loss. The sum of the iron and copper losses is not a fair basis of comparison because the sum changes as the relative value of the losses varies.

RELATION BETWEEN THE IRON AND COPPER LOSSES FOR THEIR SUM TO BE A MINIMUM

The total loss L is equal to the sum of the iron and copper losses or

$$L = L_i + L_c. \tag{32}$$

The relation between the losses as their value changes has been developed and is given in equation (28) as

$$L_c^{1/2} L_i^{1/y} = \text{constant}.$$

Substituting the value of L_c from this expression in equation (32) gives

$$L = L_i + \frac{\text{Constant}}{L_i^{2/y}}.$$

Differentiating this expression with regard to L_i gives

$$\frac{dL}{dL_i} = 1 - \frac{2}{y} \times \frac{\text{Constant}}{L_i^{(2+y)/y}} .$$

Putting this equal to zero and substituting the value of the constant from equation (32) gives

$$\frac{L_i}{L_c} = \frac{2}{y}. \tag{33}$$

If y has a value of 2, the sum of the losses will be a minimum when the iron loss is equal to the copper loss. This is a general relation and applies to transformers of any capacity, voltage or frequency, or of any type of design. If the losses bear this relation, their sum will be a minimum; whether the relative value of the losses is changed by varying the impressed voltage, keeping the output constant or by changing the number of turns in the winding, keeping the impressed voltage, current output and volume of the copper constant.

VARIATION OF THE IRON LOSS WITH CHANGE OF FREQUENCY

In the preceding cases, the losses were varied by changing the impressed voltage or the number of turns in the winding, keeping the output and the frequency constant. Another useful relation is the variation of the iron loss, keeping the voltage and output constant but varying the frequency of the current exciting the transformer. The question is one of variable iron loss only, as the copper loss is not affected by change of frequency.

From equation (20) it is apparent that the iron loss of a transformer may be expressed as follows:

$$L_i \propto f^x B^y. \tag{34}$$

Combining equations (28) and (34) gives

$$L_i \propto f^x \left(\frac{E}{fTA_i}\right)^y.$$

Since E, T and A_i are constant for a given transformer, this may be written

$$L_i \propto f^{x-y}.$$

If x be taken as 1.2 and y as 2, this equation becomes

$$L_i \propto f^{-0.8}. \qquad (35)$$

The curve in Fig. 25 shows the approximate variation of

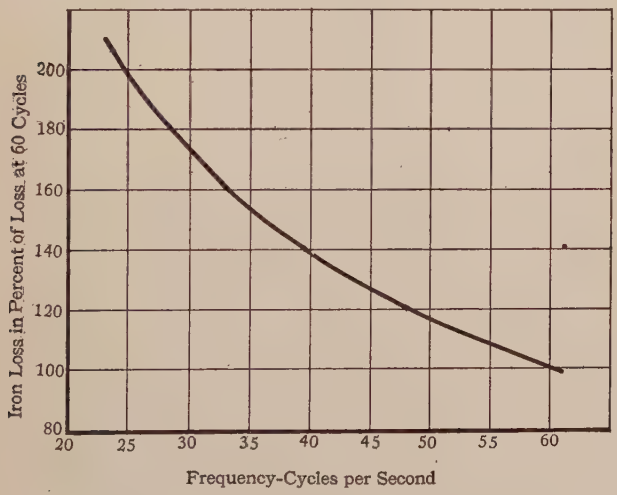


FIG. 25. Variation of the iron loss with frequency.

the iron loss of a transformer, with the frequency at which it operates, as given by equation (35).

CAPACITY RELATION

An expression for the k.v.a. output of a transformer may be developed as follows: Since the space factor S_c of a trans-

former winding is the ratio of the total net area of copper in the winding section to the gross area of the winding section A_c,

$$S_c = \frac{Q_H T_H + Q_L T_L}{A_c},$$

where Q_H and Q_L are the sectional areas of the high-voltage and low-voltage conductors respectively, and T_H and T_L are the numbers of turns in these windings.

If the current density is the same in both windings,

$$S_c = \frac{2Q_H T_H}{A_c} = \frac{2Q_L T_L}{A_c}. \tag{36}$$

Equation (29) may be put into the form

$$A_i T_L = \frac{3{\cdot}5 \times 10^6}{fB} \times E_L.$$

Substituting in this expression the value T_L from equation 36 gives

$$A_i A_c = \frac{3{\cdot}5 \times 10^6}{f} \times \frac{2Q_L E_L}{BS_c}. \tag{37}$$

For a given current density in the conductors of the winding

$$I_d Q_L = \frac{1000P}{E_L},$$

where P is the k.v.a. output of the transformer.

Substituting this value of Q_L in equation (37) gives

$$P = \frac{BI_d A_i A_c S_c f}{7 \times 10^9}. \tag{38}$$

This relation shows that the output of a transformer is directly proportional to the densities at which the iron and copper are working, the space factor of the winding section

and the areas of the magnetic and winding sections. The limit to which the densities may be increased is set for the magnetic circuit by the point of saturation of the iron and in the copper by considerations of heating. The greater these working densities, the greater are the iron and copper losses; which in certain cases may set a limit for the densities below that of saturation or overheating. When these densities become fixed a further increase in the rating of the transformer can only be secured by an increase in the space factor of the winding section, or an increase in the area of the winding or magnetic circuit sections. The limit to the increase in the space factor of the winding section is set by the necessity of providing sufficient space for the insulation and oil ducts.

Example 12: If a transformer has the following constants, what will be its output rating at 60 cycles?

$$B = 12{,}000,$$
$$S_c = 0.4,$$
$$I_d = 1{,}550,$$
$$A_i = 45,$$
$$A_o = 70.$$

From equation (38),

$$P = \frac{12000 \times 1550 \times 45 \times 70 \times 0.4 \times 60}{7 \times 10^9} = 200 \text{ k.v.a.}$$

RELATION BETWEEN OUTPUT AND FREQUENCY

Equation (38) indicates that with all factors constant except the frequency, the output is directly proportional to the frequency. For example, the output of a 60-cycle transformer on 25 cycles would be 25/60 or 42 percent of its output at 60 cycles. Under this condition of fixed magnetic induction, as the frequency is reduced the iron loss becomes less and as the frequency is increased the iron loss becomes greater.

For the condition of fixed iron loss, the watts lost per pound of iron must be constant, since the iron loss is equal

to the number of pounds of iron in the transformer times the loss per pound. By substituting in equation (38) the value of B from equation (20), and the value of I_a from equation (6),

$$P \propto W_i^{1/y} W_c^{1/2} A_i A_c S_c f^{1-(1/y)}. \tag{39}$$

With all factors in equation (39) constant except the frequency, a value of $y = 2$ gives.

$$P \propto f^{1/2}. \tag{40}$$

From this relation, the output of a 60-cycle transformer on 25 cycles, for example, is approximately 65 percent of its output at 60 cycles, the iron loss being the same in both

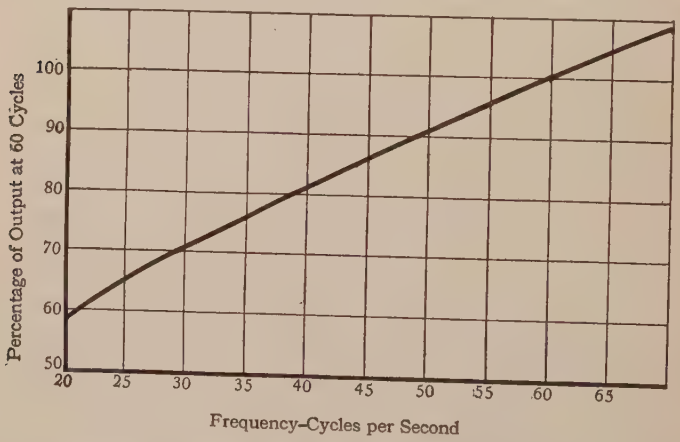


FIG. 26. Variation of output and frequency, output calculated at 60 cycles with the iron loss constant.

cases. The curve in Fig. 26 gives the output of a transformer at various frequencies in terms of its output at 60 cycles, with the iron loss constant at all frequencies.

CHAPTER V

GENERAL DESIGN RELATIONS

THERE are a number of general relations between the factors which enter into the design of a transformer which are of interest and value in showing how its various characteristics are dependent upon one another. These relations, while theoretically correct, are based upon certain assumptions which are only approximately true and for but a limited range of variation. For this reason the equations must be used with caution and with a full knowledge of the limitations imposed, or erroneous results will be secured.

RELATION BETWEEN THE OUTPUT AND DIMENSIONS, WEIGHT AND COST OF A TRANSFORMER

If in equation 38, the values of the working densities in the iron and copper, the space factor in the winding and the frequency are constant, it is evident that the output P is proportional to $A_i A_c$. Since each of these areas is proportional to the product of their dimensions, they are proportional to the square of one of the dimensions of the transformer. If one of the dimensions of the transformer be represented by b, then the product of $A_i A_c$ is proportional to b^4, then

$$P \propto b^4, \tag{41}$$

or the output of a transformer is proportional to the fourth power of its dimensions.

Example 13: If the dimensions of a transformer be increased by 10 percent, what will be the increase in its output rating, assuming that it is not limited by the heating of the windings?

From equation 41 the output of the transformer becomes $(1.1)^4$ or 1.46 times the original rating.

Since the weight and cost of the active material in a transformer increase as the cube of its dimensions, from equation 41 the relations between its output, cost and weight can be determined, as follows:

$$P \propto (\text{Weight})^{4/3}, \tag{42}$$

$$P \propto (\text{Cost})^{4/3}. \tag{43}$$

Example 14: If the cost and weight of the active material in a transformer be increased by 10 percent, what will be the increase in its output rating?

From equations 42 and 43 the output of the transformer becomes $(1.1)^{4/3}$ or 1.13 times the original rating.

The same rates of increase will apply with some degree of accuracy to the total weights and costs of the transformer, if the ranges of increase or decrease are not large and it is assumed that the same constants of design apply.

Example 15: If a 5 k.v.a. transformer weighs 235 pounds, what will be the approximate weight of a k.v.a. unit of similar design?

From equation 42 the weight of the 10 k.v.a. transformer will be approximately $235 \times (10/5)^{3/4}$ or 395 pounds.

Example 16: If the selling price of a 5 k.v.a. transformer is $73.50, what is the approximate selling price of a 10 k.v.a. unit?

From equation 43 the selling price of the 10 k.v.a. unit will be approximately $\$73.50 \times (10/5)^{3/4}$ or $123.50.

RELATION BETWEEN THE OUTPUT AND THE SUM OF THE IRON AND COPPER LOSSES OF A TRANSFORMER

Both the sum of the iron and copper losses of a transformer, as well as its total equivalent loss, may be expressed in terms of its k.v.a. output. The sum of the losses L may be expressed in terms of the iron and copper loss as follows:

$$L = L_i + L_c$$

or

$$L = W_i G_i + W_c G_c.$$

The weight of iron and copper G_i and G_c in the transformer are proportional to the volumes of those elements. Since the volume of an object is proportional to the cube of

its dimensions, if b is one of the dimensions of a transformer, the weight of both the iron and copper elements are proportional to b^3. The above equation may thus be written:

$$L \propto W_i b^3 + W_c b^3$$

or

$$L \propto (W_i + W_c) b^3.$$

Assuming that the flux density in the iron and the current in the copper are constant, this relation may be written:

$$L \propto b^3. \tag{44}$$

That is, the sum of the losses varies as the cube of the linear dimensions of the transformer. It has been shown in equation 43 that $P \propto b^4$ and since $L \propto b^3$

$$L \propto (P)^{3/4}. \tag{45}$$

This means that with transformers of greater and greater size, using more copper and iron in their coils and magnetic circuits, the output and total loss increase such that the sum of the losses increases as the $\frac{3}{4}$ power of the output rating.

Example 17: If the sum of the losses for a 25 k.v.a. transformer is 570 watts, what will be the approximate sum of the losses for a 50 k.v.a. unit of similar design?

From equation 45 the sum of the losses for the 50 k.v.a. transformer will be approximately $570 \times (50/25)^{3/4}$ or 958 watts.

If equation 45 be divided through by P, the following exression results, which gives the relation between the total losses for different sizes of transformers in a form more convenient for ordinary use:

$$\text{Percentage of total loss} \propto \frac{1}{P^{1/4}}. \tag{46}$$

Example 18: If the percentage of total loss of a 7.5 k.v.a. transformer is 2.67 percent, what is the approximate percentage of total loss for a 15 k.v.a. unit?

From equation 46 the loss for the 15 k.v.a. transformer would be approximately $2.67 \times (7.5/15)^{\frac{1}{4}}$ or 2.25 percent.

RELATION BETWEEN THE OUTPUT AND TOTAL EQUIVALENT LOSS OF A TRANSFORMER

Equation 39 may be written, when the frequency is constant, as follows:

$$W_i^{1/y} W_c^{1/2} \propto \frac{P}{A_i A_c S_c}.$$

Multiplying through by $G_i^{1/y} G_c^{1/2}$, where G_i and G_c are the weights of iron and copper respectively, gives

$$L_i^{1/y} L_c^{1/2} \propto \frac{P G_i^{1/y} G_c^{1/2}}{A_i A_c S_c},$$

or when y is equal to 2

$$L_i L_c \propto \left(\frac{P G_i^{1/y} G_c^{1/2}}{A_i A_c S_c} \right)^2. \tag{47}$$

This equation shows the relation between the equivalent total loss of the transformer and its output rating. Assuming a transformer of fixed dimensions, working at a definite frequency, the equivalent total loss is proportional to the square of the output rating. That is,

$$L_i L_c \propto P^2. \tag{48}$$

Example 19: If the product of the losses for a 100 k.v.a. transformer is 6×10^5, what is the product of the losses for a 200 k.v.a. unit?

From equation 48 the product of the losses for the 200 k.v.a. transformer will be approximately $6 \times 10^5 \times (200/100)^2$ or 24×10^5. If the ratio of the copper to the iron loss is 2 for the 100 k.v.a. unit it will be the same for the 200 k.v.a. transformer, therefore,

$$L_i = \sqrt{\frac{24 \times 10^5}{2}} = 1095 \text{ Watts},$$

$$L_c = 2 \times 1095 = 2190 \text{ Watts}.$$

Equation 48 may be divided through by P, which gives

$[(L_i L_c)/P] \propto P$, or the equivalent total loss expressed as a percentage of the output is proportional to the output rating or

$$\text{Percentage of equivalent total loss} \propto P. \qquad (49)$$

This means that if a curve be plotted between the percentage of equivalent total loss and output ratings, it will be a straight line if the same constants of design be maintained, and the ratio of the copper and iron losses is constant. This relation gives an easy way to determine the losses of a line of transformers, when the losses for two similar designs of different rating are known.

Example 20: If the iron and copper losses for a 5 and a 200 k.v.a., 2,300 to 230 volt, 60-cycle transformer, are as given in the table below, what are the losses of the other sizes of the line using the same constants of design?

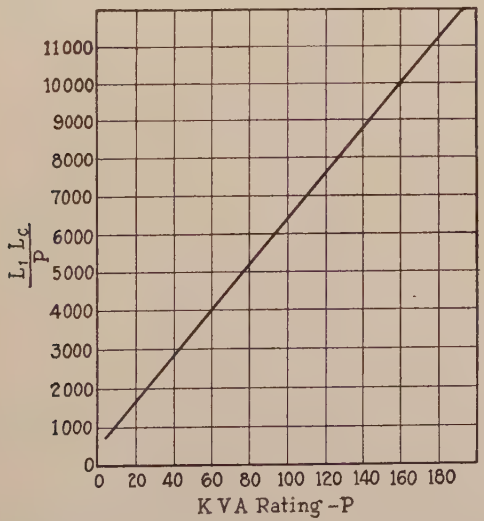


FIG. 27. The equivalent total loss expressed as a percentage of the transformer k.v.a. rating directly proportional to the k.v.a. rating.

K. V. A.	Watts Iron Loss	Watts Copper Loss by Wattmeter at 75° C.	$\frac{L_i L_c}{P}$
5	45	90	810
200	1120	2240	12500

Plotting the curve between k.v.a. rating and $(L_i L_c)/P$ gives the straight line shown in Fig. 27. The performance of this line of transformers from this curve would be approximately as follows:

K. V. A.	Watts Iron Loss	Watts Copper Loss by Wattmeter at 75° C.
5	45	90
7.5	61	122
10	74	148
15	99	198
25	158	316
37.5	225	450
50	304	608
75	433	866
100	570	1,140
125	710	1,420
150	850	1,700
200	1,120	2,240

RELATION BETWEEN THE TOTAL EQUIVALENT LOSS AND THE DIMENSIONS, WEIGHT AND COST OF A TRANSFORMER

If in equation 47 the output and space factor remain constant and the size of the transformer be changed by varying its dimensions, the equivalent total loss may be expressed in terms of the dimensions, as follows:

$$L_i L_c \propto \left(\frac{1}{b}\right)^2. \qquad (50)$$

This follows from the facts that the areas of the magnetic circuit and the winding sections are proportional to the square of b, the weights of iron and copper to the cube of b, and that y is equal to 2.

Example 21: If the dimensions of a transformer be increased 10 percent, what will be the change in the equivalent total loss?

From equation 50 the equivalent total loss becomes $(1/1.1)^2$ or 0.83 times the original value.

Further, since the weight and cost of the active material varies as b^3 this expression may be written:

$$L_i L_c \propto \left(\frac{1}{\text{Weight}}\right)^{2/3} \tag{51}$$

and

$$L_i L_c \propto \left(\frac{1}{\text{Cost}}\right)^{2/3}. \tag{52}$$

Example 22: If the weight and cost of the active material in a transformer be increased 10 percent, what will be the change in the equivalent total loss?

From equations 51 and 52 the equivalent total loss becomes $(1/1.1)^{2/3}$ or 0.93 times the original value.

If the copper loss is constant,

$$\text{Cost} \propto \left(\frac{1}{L_i}\right)^{3/2}, \tag{53}$$

and when the iron loss is constant,

$$\text{Cost} \propto \left(\frac{1}{L_c}\right)^{3/2}. \tag{54}$$

Example 23: If the iron loss of a transformer be reduced 10 percent, how much can the cost of its active material be reduced, the copper loss remaining constant?

From equation 53 the cost of the active material becomes $(1/0.9)^{3/2}$ or 1.18 times the original value.

RELATION BETWEEN THE COST OF THE ACTIVE MATERIAL FOR A GIVEN IRON LOSS AND THE QUALITY OF IRON IN THE MAGNETIC CIRCUIT

For a constant iron loss, if the quality of the iron in the magnetic circuit be improved, less material may be used. Equation 53 gives the relation between the iron loss and the cost of the active material but, if the percent quality of the iron be improved, the result is equivalent to increasing

the iron loss of the transformer the same percentage, therefore,

$$\text{Cost} \propto (\text{percent iron})^{3/2}. \qquad (55)$$

This assumes that the magnetic density in the iron is not limited by its saturation, but that it may be freely varied through the range permitted by the change in the quality of the iron.

Example 24: What would be the relative costs of two transformers, one built with 100 percent quality of iron and one built with 90 percent quality of iron?

From equation 55 the cost of the transformer with 90 percent iron is $(0.90)^{3/2}$ or 0.85 of the cost of the transformer with 100 percent iron.

VARIATION OF THE COST WITH THE SPACE FACTOR OF THE WINDING SECTION

With everything constant except the dimensions of the transformer and the space factor of the winding section, it will be found from equation 47, remembering that the cost of the active material varies as b^3, that

$$\text{Cost} \propto (1/S_c)^3. \qquad (56)$$

Example 25: What will be the relative cost of the active material in two transformers, one of which has a ten percent better winding space factor than the other?

From equation 56, the cost of the transformer with the better space factor is $(1/1.10)^3$ or 0.75 of the cost of the transformer having the poorer space factor.

CHAPTER VI

GENERAL PROBLEMS OF DESIGNS

In the usual discussion of transformer design, the mathematical difficulties often obscure the actual relations between the various quantities involved. The following analysis is intended to show these relations in a general way without the use of mathematics. It is obvious that the relations between all of these variables mathematically developed would result in rather complicated operations and expressions.

TYPES OF CONSTRUCTION

The shell and core types are the two fundamental forms of transformer construction. All others are modifications of these. In the former, the iron encloses the winding like a

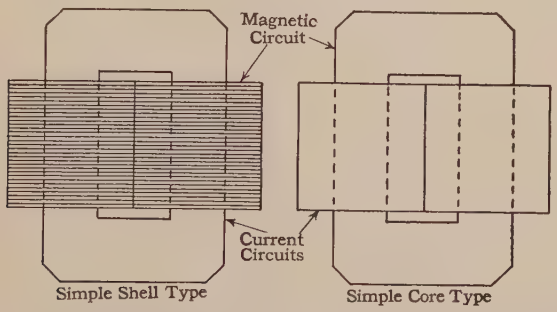


Fig. 28. Fundamental relation between shell- and core-type of transformer construction.

shell, and in the latter the iron forms the core upon which the windings are mounted. The relation between these fundamental types is given in Fig. 28, which shows that an

interchange of the iron and copper elements will transform the shell type into the core type, and vice versa.

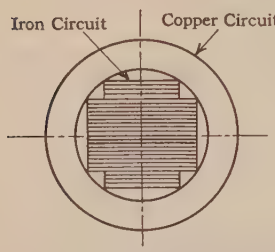


FIG. 29. Section through one side of a core-type transformer.

A section through the copper element at right angles to the conductors is usually rectangular in shape for both the shell and core types, which also gives a rectangular section through the iron element. Sometimes the section of the iron circuit is given a cruciform shape in which case the windings take a circular form as shown in Fig. 29. This cruciform design is used mainly for the high voltage core type transformers because of the ease with which the circular coils may be wound and because sharp bends in the insulation are avoided.

The development of the distributed shell and core types of construction from the simple forms is by the addition of two smaller iron circuits to the shell type, or two small copper circuits to the core type, as shown in Fig. 30. The reason

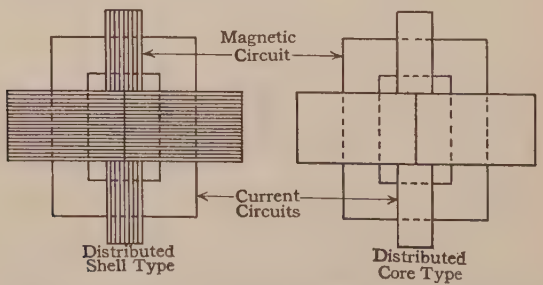


FIG. 30. Relation between the distributed shell- and core-type transformer construction.

for the use of this form is the economy secured by the use of the smaller mean turn of the element which is separated or distributed into a number of parts.

In the shell type of design in all its forms and in the simple core type, it is possible to widen those portions of the magnetic circuit which are not surrounded by the windings, as shown in Figs. 28 and 29. An increase in the area of these portions of the magnetic circuit permits them to run at a lower magnetic density than the unwidened parts, thus giving a lower iron loss. When low losses are desired better results can usually be obtained by this expedient than by putting the increased material into a uniformly increased section of either iron or copper.

Another modification of the core construction is shown in Fig. 31 which is the form used for three-phase transformers. There are three separate magnetic circuits, one for each of the three-phase fluxes. Such a construction produces complications in the assembly of the various parts, which are

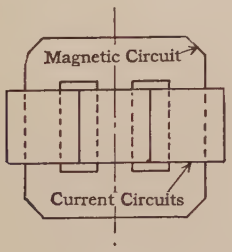


FIG. 31. Three-phase core-type transformer.

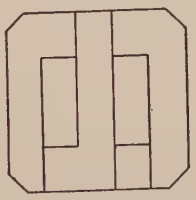


FIG. 32. Arrangement of the laminations.

overcome as shown in Fig. 32. While a three-phase transformer can be built with three separate magnetic circuits, the only saving as compared with three single-phase transformers is the use of a single case as against the use of three with the single-phase units. With the three magnetic circuits interlinked, there is a saving in material in comparison with three single-phase transformers aggregating the same capacity.

SELECTION OF TRANSFORMER TYPE

The shell and core types of transformer construction each have a fairly well marked portion of the total range of k.v.a., voltage and frequency, to which it is best adapted, other conditions being the same. The most direct way to make a comparison of the relative economy of the two types for a particular transformer rating is to make comparative designs for both types, the comparison being usually made on the basis of a minimum cost of material required for a transformer which will meet given insulation tests, and to have certain performance characteristics. Under analysis it is found, on the basis of economy of material alone, that small, high-voltage transformers should be core type and large, low-voltage transformers should be shell type. Further it is found that the main factor which controls the design, as to type, is the space factor of the winding, or the percentage of the total winding space occupied by copper. The space not occupied by copper is that required for insulation or for ventilating ducts. Small high-voltage transformers have a relatively poor space factor, which grows better as the k.v.a. increases and the voltage rating decreases. While the relation between the space factor of the winding and the most economical type of transformer to meet certain specifications is not obvious, the following explanation will make it more apparent.

A shell type of transformer has a short mean turn of iron, while a core type has a short mean turn of copper. There is a tendency to use the greater amount of the material having the shorter mean turn, because with a given amount of material, the part which has the short mean turn is the most effective in producing an element of large sectional area. The materials are economically disposed when, with a given amount of iron and copper, the sectional area of each is a maximum. This is true because the output of the transformer is a maximum when the product of these areas is a

maximum. With the shell type of transformer, since the weight of copper is small, the sectional area is small and the space factor of the cpoper has a great effect upon the design; that is, the winding with a large percentage of insulation is at a disadvantage with this type. On the other hand, with a core type of transformer, whose weight of copper is relatively large, greater constant insulation values do not affect the design so markedly.

For each voltage and frequency class, therefore, there is a transformer of a certain output rating, above which the space factor makes the shell type cheaper and below which the core type uses less material.

EFFECT OF CHANGING THE PROPORTIONS OF A TRANSFORMER

A transformer of fixed output and dimensions was considered in chapter IV, and the losses were varied by changing the densities of working the iron and copper. Since the dimensions of the iron and copper elements remained constant, the weight of these parts did not change. In another case the iron loss only was made to vary by changing the frequency at which the transformer was working.

The problem of design involves varying the losses by changing the dimensions and weight of the magnetic circuit and windings, which means that the cost of the materials also changes. This condition can best be analyzed by the use of equation 38. For this purpose a slight modification of this equation may be made by representing N the product of the areas of the iron section A_i and of the winding section A_c or

$$N = A_i A_c.$$

Substituting this value in equation 38 gives

$$P = \frac{BI_d S_c f N}{17 \times 10^9}.$$

Keeping the output, working densities, frequency and winding space factor constant, the value of N in this equation becomes constant. This means that the product of the area of the winding space and the area of the iron section is fixed, but that the values of the areas themselves may be variable. This allows a variation in the dimensions of the transfromer, as for instance A_c may be large in which case, A_i being small, the weight of copper in the transformer would be relatively greater than the weight of iron. On the other hand, A_i might be larger and the transformer would be relatively heavy in iron. Since the energy loss per pound of iron and copper remains constant, the losses will vary as the weights of these materials in the magnetic circuit and windings. In addition to the changes resulting from a variation in the relative values of A_i and A_c, the relative dimensions of the iron and copper sections may be changed. There are, therefore, three main variables to be taken into account:

1. The ratio of the area of the iron to the area of the copper section.
2. The ratio of the dimensions of the winding section.
3. The ratio of the dimensions of the iron section.

The distributed shell or core type introduces another variable, which is the ratio of the areas of the two small circuits to the areas of the two larger ones. Also widening of the magnetic circuit introduces a fifth variable into the design, which is the ratio of the widened section to the unwidened part, thus making five variables in the most general case of transformer design.

EFFECT OF CHANGING THE RATIO OF THE AREA OF THE IRON TO THE AREA OF THE COPPER SECTION

This ratio is the most important of the five variables which are involved in the most general case of design, as a small

change in its value has a marked effect on the losses and cost. As the area of the iron section increases and that of the copper section decreases, the weight of iron, the iron loss and the cost of the iron increases and the weight of copper, copper loss and cost of copper decreases. If the copper section has a low space factor, that is, a small amount of copper, a reduction of the area of the copper section will not mean a large reduction in weight or cost of the copper. Therefore, with a low copper space factor the iron section will be greater and the gross copper section will be larger than with a high copper space factor. The actual value of the variable in a particular case depends on the relative cost of iron and copper per pound, as well as the insulation clearances or the space factor of the copper section.

EFFECT OF CHANGING THE RATIO OF THE DIMENSIONS OF THE IRON AND COPPER SECTIONS

It is evident that the length of the mean turn of both the iron and the copper elements should be a minimum for a given area of their sections. Thus when the mean length of turn of each element is a minimum, a given number of turns of copper would have a minimum weight and resistance, and the iron circuit would have a minimum amount of material. For example, with the iron section a square, the mean turn of copper would be a minimum for a given area of the iron section, but on the other hand, the mean turn of the iron itself would be greater than if the iron section were rectangular. It is therefore apparent that there is a direct relation between these two variables.

Small changes in these variables, however, have only a slight effect on the design and when their best values have been determined they may thereafter be regarded as constants without appreciable loss, unless there is some other reason for a change in their value than the question of economy of material or reduction in loss.

RATIO OF THE AREAS OF THE LARGE AND SMALL CIRCUITS WITH THE DISTRIBUTED SHELL AND CORE TYPES

Since the gain by the use of the distributed shell and core types of construction, as compared to the simple forms, is not great, slight changes in this variable would not be expected to affect the design materially. When its value has once been determined it may thereafter be considered a constant, even though theoretically it is a variable whose value depends on the ratio of the dimensions of the winding section, which in turn is related to the other design variables.

PERCENTAGE OF WIDENING OF THE MAGNETIC CIRCUIT

The widening of the magnetic circuit may be looked upon as an expedient to reduce the iron loss by reducing the magnetic density in the widened part. The loss in the widened portion will decrease approximately as the square of the increase in area, but the weight of iron in this part will increase by an amount proportional to the increase of the area, so that the iron loss in the widened part will decrease approximately as the percent widening. A similar reduction in iron loss can be secured by increasing the dimensions of the transformer as a whole, in which case the cost would increase at a different rate than when the cost is increased by widening. The relation between these two rates of cost increase is a complicated one. It is found that when a certain widening is reached, it is cheaper to further reduce the iron loss by putting more material into the transformer as a whole than by additional widening. The most economical widening is different for the different types of transformers, being greater for the shell than for the core type.

Another reason for the use of the widened magnetic circuit is that the exciting current is reduced, partly by putting the joints in the low density part and because ordi-

narily the exciting current is reduced by the same steps which reduce the iron loss. While theoretically the percentage widening is related to the other design variables, practically it may be considered as a constant after a reasonable value has once been determined upon for some particular case.

METHODS OF DESIGN

There are two fundamental premises from which the design of a transformer may be started. These are as follows:

1. With the magnetic and current densities fixed, to design a transformer having a minimum cost.
2. With the magnetic and current densities fixed, to design a transformer for which the sum of the losses will be a minimum.

The first premise is to make the cheapest possible design without considering the losses. Because the copper cannot be worked beyond a certain density because of heating, and the iron cannot be operated beyond a definite magnetic density because of saturation of the circuit, there is a limit to the cheapness of a transformer for a given rating. Thus the problem becomes one of designing a transformer with fixed current and magnetic densities. For this case where low losses are not important, it would not be necessary to use either the distributed shell or core types of design, or to widen the magnetic circuit. The proper design for this case will usually be the one whose cost of iron and copper are approximately equal.

To design a transformer for a minimum sum of the losses instead of a minimum cost, with the densities fixed in both cases, the correct design would usually be the one where the iron loss is approximately equal to the copper loss. The designs for the two cases would be identical if the ratio of the cost per pound of iron to the cost per pound of copper is equal to the ratio of the watts lost per pound in each.

When a transformer is designed to have given losses, the problem is not primarily one of fixed densities, because in this case the densities depend on the given losses. For example, if the iron loss is to be small, the iron density must be correspondingly low, and the same condition applies to the copper circuit. Therefore, if the iron and copper densities are low to give small losses, the weights of iron and copper will be correspondingly high and the cost of the transformer will be large. To view this last problem from another angle, suppose a transformer was designed with the magnetic and current densities fixed and for a given equivalent total loss. If this equivalent loss is for example larger than the equivalent loss desired, the magnetic and copper densities must be reduced. If now the relative value of the iron and copper losses is not that desired, they may be varied in the manner described in chapter IV. In case the saturation of the magnetic circuit or heating of the copper circuit will not permit the desired relation between the iron and copper losses to be secured, the design must be distorted in order to secure this result. If for example the iron loss is not low enough, the iron section must be made smaller and the copper section larger until the reduced amount of iron will permit the desired iron loss to be secured.

CHAPTER VII

AN APPROXIMATE METHOD OF DESIGN

THE cases of design outlined in chapter VI can be carried out in detail for various types of transformers. The treatment here given is not complete and is intended to be more of a suggestion than of an exhaustive nature. For the sake of simplicity it is limited as follows:

1. Simple shell and core types with rectangular iron and copper sections.
2. Unwidened magnetic circuit.
3. Copper uniformly distributed over the winding space.

The last assumption is approximately true only, as in reality there are fixed insulation clearances, depending on the voltage between the windings and the iron which influence the design to a considerable extent; particularly for the higher voltages.

CAPACITY CONSTANT

The most obvious point from which to start the design of a transformer is to determine the capacity constant N, which is the product of the areas of the iron section A_i and the copper section A_c from equation 38. This equation is

$$N = A_i A_c = \frac{7 \times 10^9 \times P}{B I_d S_i S_c f}. \qquad (57)$$

where P is the k.v.a. rating of the transformer, B is the flux density in the magnetic circuit, I_d is the current density in the copper circuit, S_i is the space factor of the iron circuit, S_c is the space factor of the copper circuit, and f is the frequency of the current.

The values of B and I_d chosen depend upon the iron and copper losses desired and their upper limit is set by saturation of the iron and heating of the copper. Low values for these densities will give low iron and copper losses, and the transformer will have a relatively large amount of material, and therefore be expensive. The value of S_e depends on the capacity rating, voltage and frequency of the transformer.

Example 26: What is the capacity constant of a 2,300 to 230 volt transformer which has the following characteristics? $P = 200$; $B = 12{,}000$; $I_d = 1{,}550$; $S_c = 0.4$; $S_i = 1.0$ $f = 60$.

From equation (57),

$$N = \frac{7 \times 10^9 \times 200}{12000 \times 1550 \times 0.4 \times 60} = 3150.$$

DIMENSIONS OF THE TRANSFORMER

Even with a fixed value of N, the transformer is not designed until values are determined for A_i, A_c, b_c, a_c, b_i and a_i, when

$$A_i = b_i\, a_i, \qquad (58)$$

$$A_c = b_c\, a_c, \qquad (59)$$

and b_c, a_c, b_i and a_i are the dimensions of the copper and

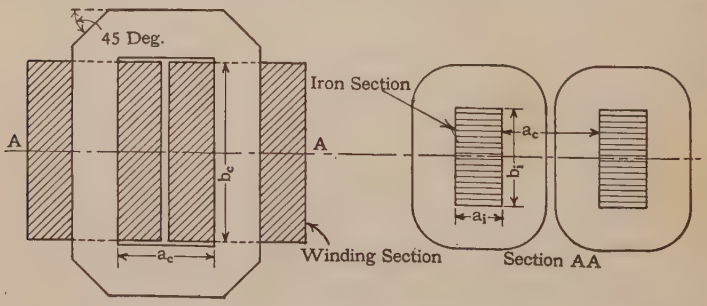


FIG. 33. Core-type transformer with a section through the winding at right angles to the magnetic circuit.

iron circuits, as shown in Figs. 33, 34 and 35. These six variables may be reduced to three as follows:

Let

$$X^2 = \frac{A_i}{A_c}, \tag{60}$$

$$Y^2 = \frac{b_c}{a_c}, \tag{61}$$

$$Z^2 = \frac{b_i}{a_i}. \tag{62}$$

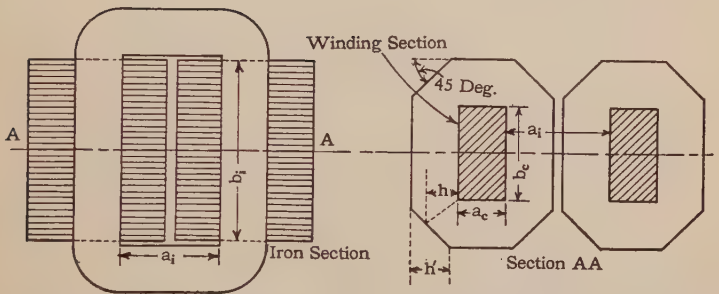


FIG. 34. Shell-type transformer with a section through the winding at right angles to the magnetic circuit.

X^2, Y^2 and Z^2 are used for these ratios instead of X, Y and Z, because of the simplification this makes in the following algebraic work. The sectional areas and dimensions of the tranformer may be written as follows, by the help of equations (57), (58), (59), (60), (61) and (62),

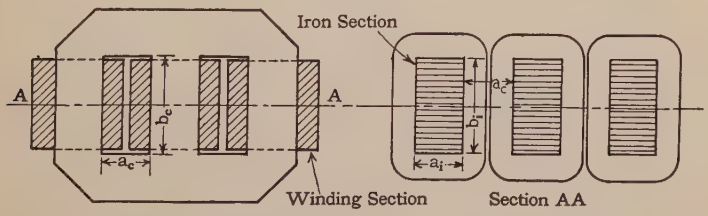


FIG. 35. Core-type three-phase transformer with a section through the winding at right angles to the magnetic circuit.

$$A_i = (X^2 N)^{1/2}, \tag{63}$$

$$A_c = \frac{N}{A_i}, \tag{64}$$

$$b_c = (Y^2 A_c)^{1/2}, \tag{65}$$

$$a_c = \frac{b_c}{Y^2}, \tag{66}$$

$$b_i = (Z^2 A_i)^{1/2}, \tag{67}$$

$$a_i = \frac{b_i}{Z^2}. \tag{68}$$

Example 27: What are the values of A_i, A_c, b_c, a_c, b_i and a_i for the transformer covered by the preceding example of the shell type of design, whose value of N is 3,150, when $X^2 = 2.1$; $Y^2 = 2.3$; $Z^2 = 2$?

From equations (63) to (68),

$$A_i = (2.1 \times 3150)^{1/2} = 81.4 \text{ sq. in.}$$

$$A_c = \frac{3150}{81.4} = 38.7 \text{ sq. in.}$$

$$b_c = (2.3 \times 38.7)^{1/2} = 9.44 \text{ in.}$$

$$a_c = \frac{9.44}{2.3} = 4.1 \text{ in.}$$

$$b_i = (2 \times 81.4)^{1/2} = 12.75 \text{ in.}$$

$$a_i = \frac{12.75}{2} = 6.37 \text{ in.}$$

EXPRESSIONS FOR THE MEAN TURN OF IRON AND COPPER

The mean turn of the iron or copper elements is that quantity which, multiplied by its sectional area, gives its total volume. The mean turn of iron for the shell type is

$$l_i = 2(a_c + b_c + 0.828\ a_i). \tag{69}$$

where a_c, b_c and a_i are the dimensions of the circuits as shown in Fig. 34. Assuming that the corners of the

magnetic circuit are cut off at an angle of 45 degrees, tangent to a radius of $a_i/2$, the constant 0.828 is determined as follows:

From Fig. 34

$$h = \frac{a_i}{2} \cos 45 \text{ degrees},$$

$$\frac{h'}{2} = \frac{a_i}{2} - \frac{a_i}{2} \cos 45 \text{ degrees}$$

or

$$h' = 0.586 \frac{a_i}{2}.$$

$$\text{Area of one corner} = \left(\frac{a_i}{2}\right)^2 - \frac{1}{2}\left(0.586 \frac{a_i}{2}\right)^2.$$

This expression divided by $a_i/2$ gives

$$\text{Mean turn of corner} = \frac{0.828 a_i}{2}.$$

And the total mean turn is therefore that given by equation (69).

If the corners of the copper circuit are rounded with a radius of a_c, as shown in Fig. 34,

$$l_c = 2\left(a_i + b_i + \frac{\pi}{2} a_c\right) = 2(a_i + b_i + 1.57 a_c).$$

In the expression for the mean turn of iron for the shell type, equation (69), replace $a_i/2$ by a_i, which gives for the mean turn of the core type

$$l_i = 2(a_c + b_c + 1.656\ a_i).$$

In case the corners of the windings are rounded with a radius of $a_c/2$, as shown in Fig. 33,

$$l_c = 2\left(a_i + b_i + \frac{\pi}{4} a_c\right) = 2(a_i + b_i + 0.785 a_c).$$

For convenience in the algebraic work following, the mean turn of iron and copper may be expressed in general terms as follows, applying to either the shell or core type,

$$l_i = 2(a_c + b_c + c'a_i), \qquad (70)$$
$$l_c = 2(a_i + b_i + ca_c), \qquad (71)$$

where c' and c are constants which have the following values:

	c'	c
Shell type	0.828	1.57
Core type	1.656	0.785

GENERAL EXPRESSIONS FOR THE WEIGHTS OF IRON AND COPPER

From the mean turn expressions given by equations (70) and (71), the weights of iron and copper are as follows:

$$G_i = 2\ D_i\ S_i\ A_i\ (a_c + b_c + c'a_i), \qquad (72)$$
$$G_c = 2\ D_c\ S_c\ A_c\ (a_i + b_i + ca_c). \qquad (73)$$

where D_i is the number of pounds per cubic inch of the magnetic circuit, S_i the space factor of the iron, A_i the gross area of the iron section, and D_c, S_c and A_c the corresponding quantities for the copper circuit. For sheet steel as in curve, Fig. 10, $D_i = 0.244$, and the pounds per cubic inch of copper, $D_c = 0.321$.

Example 28: What are the weights and costs of iron and copper, and the iron and copper losses for the shell type transformer covered by the preceding examples?

From equations (72) and (73), the weights of iron and copper are:

$$G_i = 2 \times 0.244 \times 81.4(4.1 + 9.44 + 0.828 \times 6.37) = 748 \text{ lbs.},$$
$$G_c = 2 \times 0.321 \times 0.4 \times 38.7(6.37 + 12.75 + 1.57 \times 4.1) = 254 \text{ lbs.}$$

A density of 12,000 gausses gives from the curve in Fig. 10 a value for the watts per pound W_i of 1.58. A current density of 1,550 amperes per square inch at 75° C. gives from equations 3 and 6 a loss per pound W_c of 6.21. The iron and copper losses therefore are:

$$L_i = 1.58 \times 748 = 1{,}182 \text{ watts},$$
$$L_c = 6.21 \times 254 = 1{,}580 \text{ watts},$$
$$L_i + L_c = 1{,}182 + 1{,}580 = 2{,}762 \text{ watts}.$$

The cost of active material based on 15 cents per pound of iron and 45 cents per pound of copper in the transformer are:

$$C_i = 0.15 \times 748 = \$112.20,$$
$$C_c = 0.45 \times 254 = \$114.30,$$
$$C_i + C_c = 112.2 + 114.3 = \$226.50.$$

TABLE I

TABULATED TRANSFORMER DESIGN

	Shell Type Single-Phase	Core Type Single-Phase	Core Type Three-Phase
P—K.v.a.	200	200	200
f—Cycles per sec.	60	60	60
I_d—Amp. per sq. in.	1550	1550	1550
B—Gausses	12000	12000	12000
S_i—	1.0	1.0	1.0
S_c	0.4	0.4	0.36
N	3150	3150	4620
X^2	2.1	1.0	0.4
Y^2	2.3	2.3	2.7
Z^2	2	2	2
A_i sq. in.	81.4	56.2	43
A_c sq. in.	38.7	56.2	53.7
b_c in.	9.44	11.37	11.5
a_c in.	4.1	4.94	4.25
b_i in.	12.75	10.6	9.3
a_i in.	6.37	5.3	4.65
G_i lbs.	748	705	800
G_c lbs.	254	285	322
L_i watts	1182	1115	1270
L_c watts	1580	1770	2010
$L_i + L_c$ watts	2762	2885	3280
C_i dollars	112.2	106	120
C_c dollars	114.3	128	145
$C_i + C_c$ dollars	226.5	234	265

Example 29: Suppose it is desired to design a core type transformer, based upon the same assumptions as the preceding examples for the shell type, except the value of X^2 which is 1.0.

These two designs for the shell and core types are tabulated in Table I, showing the detailed relations between them. These designs are not necessarily the best possible or even practical ones, but are given only to illustrate the plan of design outlined, which is based upon certain assumptions.

THREE-PHASE TRANSFORMERS

Since in a three-phase, core-type transformer there are two openings in the iron, as in Fig. 35, instead of one for the single-phase, equation 57 must be written:

$$N = 2A_iA_c = \frac{9.32 \times 10^9 \times P}{BI_dS_iS_cf}. \qquad (74)$$

For the same reason,

$$X^2 = \frac{A_i}{2A_c}. \qquad (75)$$

Equation 64 then becomes

$$A_c = \frac{N}{2A_i}. \qquad (76)$$

The mean turn of iron for the three-phase transformer equals

$$2(2a_c + 1.5b_c + 2.65a_i). \qquad (77)$$

The weight of iron equals

$$2D_iS_iA_i(2a_c + 1.5b_c + 2.65a_i). \qquad (78)$$

The weight of copper equals

$$2 \times 1.5 \times D_cS_cA_c(a_i + b_i + 0.785\ a_c). \qquad (79)$$

Example 30: Design a 200 k.v.a., 2,300 to 230 volt, 60-cycle, three-phase transformer which has the following constants,—$B = 12{,}000$; $I_d = 1{,}550$; $S_c = 0.36$; $S_i = 1.0$; $X^2 = 0.4$; $Y^2 = 2.7$; $Z^2 = 2$.

This design is shown tabulated in Table I.

CHAPTER VIII

EFFICIENCY

By DEFINITION the efficiency of a transformer is the ratio of its energy output to the energy input; or in other words, of the output to the output plus the losses. Unless otherwise stated, the efficiency of a transformer is based on a load of 100 percent power-factor; being in this case the ratio of the k.v.a. output to the k.v.a. output plus the losses.

CALCULATION OF EFFICIENCY

By definition the efficiency of a transformer at full load is

$$\eta = \frac{P}{P + L_i + L_c}. \tag{80}$$

where the output and losses are expressed in watts.

Example 31: What is the efficiency of a five k.v.a. transformer whose iron loss is 42 watts and whose copper loss is 85 watts? From equation (80),

$$\eta = \frac{5000}{5000 + 42 + 85} = 0.9752 \text{ or } 97.52 \text{ percent.}$$

Equation (80) may be written for three-fourths load as follows, remembering that the copper loss varies as the square of the output.

$$\eta_{3/4} = \frac{\frac{3}{4}P}{\frac{3}{4}P + L_i + (\frac{3}{4})^2 L_c}. \tag{81}$$

Example 32: What is the efficiency at ¾ load of a five k.v.a. transformer, which has an iron loss of 42 watts and a copper loss of 85 watts? From equation (81),

$$\eta_{3/4} = \frac{\frac{3}{4} \times 5000}{\frac{3}{4} \times 5000 + 42 + (\frac{3}{4})^2 85} = 0.9765 \text{ or } 97.65 \text{ percent.}$$

In determining efficiencies by the use of equations (80) and (81), if accurate results are wanted, rather long numerical calculations are necessary. This may be avoided by the use of the following method, which gives good results when the total loss does not exceed about five percent. Equation (80) may be put into the form

$$\eta = \left(1 + \frac{L_i + L_c}{P}\right)^{-1}.$$

This, expanded by the binomial theorem, gives

$$\eta = 1.0 + \left(\frac{L_i + L_c}{P}\right)^2 - \frac{L_i + L_c}{P}. \qquad (82)$$

Example 33: What is the efficiency of a five k.v.a. transformer which has an iron loss of 42 watts and a copper loss of 85 watts? From equation (82),

$$\eta = 1.0 + \left(\frac{42 + 85}{5000}\right)^2 - \frac{42 + 85}{5000} = 0.9752 \text{ or } 97.52 \text{ percent.}$$

Equation (82) may be written for any other load, remembering that the copper loss varies as the square of the output. For example,

$$\eta_{3/4} = 1.0 + \left[\frac{L_i + (3/4)^2 L_c}{3/4 P}\right]^2 - \frac{L_i + (3/4)^2 L_c}{3/4 P}$$

or

$$\eta_{3/4} = 1.0 + \left(\frac{4/3 L_i + 3/4 L_c}{P}\right)^2 - \frac{4/3 L_i + 3/4 L_c}{P}. \qquad (83)$$

Equation (83) shows that when obtaining efficiencies by the use of equation (82) at other than full load, the percent iron and copper losses must be multiplied by factors depending on the load. These factors for the various loads are given in Table II.

Example 34: What is the efficiency at 3/4 load of a five k.v.a. transformer

having 42 watts iron loss and 85 watts copper loss? From equation (83),

$$\eta = 1.0 + \left(\frac{1.333 \times 42 + 0.75 \times 85}{5000}\right)^2 - \frac{1.333 \times 42 + 0.75 \times 85}{5000} = 97.65.$$

TABLE II

CORRECTION FACTORS

Load	Iron Loss Factor	Copper Loss Factor
1½	0.666	1.5
1¼	0.8	1.25
1	1.0	1.00
¾	1.333	0.75
½	2.0	0.5
¼	4.0	0.25

Ordinary slide rule calculations will give more accurate results by the use of equation (83) than when based on equation (80).

The use of Table III will sometimes be helpful when calculating efficiencies. This table gives the efficiency, when the total percent loss is given.

Example 35: By the use of Table III, find the efficiency of a five k.v.a. transformer which has an iron loss of 42 watts and a copper loss of 85 watts. The total loss is 127 watts, or 2.54 percent of 5,000 watts. Under the percent loss column at the left opposite 2 percent, go across the table to the right. Interpolating between columns headed .5 and .6 for .54 gives an efficiency of 97.52.

When a large number of efficiency calculations are to be made, charts may be prepared which give the efficiency at several loads when the percentage iron and copper losses are known. The method of using such a chart is evident from an inspection of Fig. 36, which consists of six vertical scales for showing efficiencies and three vertical scales for the losses. For each transformer there exists a relation between the points on these scales, such that when any two points are located, any others can be found.

To read efficiencies with any given losses, a straight-edge should be laid on the chart so as to intersect the vertical line at the extreme right representing the percent iron loss at the given point and at the same time to intersect the

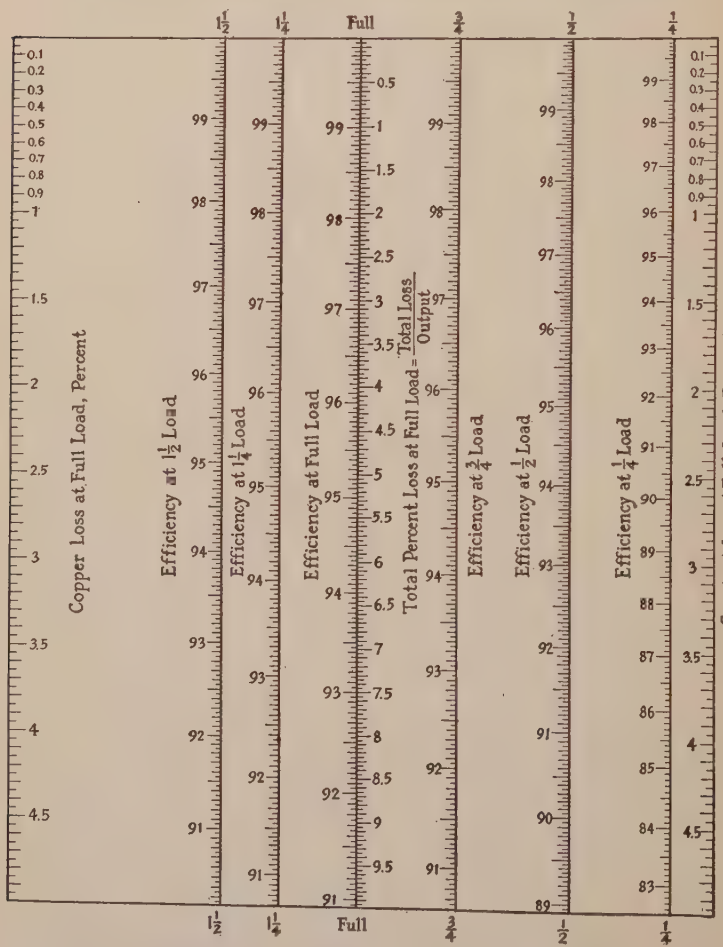


Fig. 36. Efficiency chart. To obtain efficiency, lay a straight edge across the given copper loss and constant loss values. The efficiencies at the required loads and the total percent loss at full load can be obtained simultaneously.

vertical line representing copper loss or the one representing total loss at the given percent. When this setting is made the straight-edge will intersect the other vertical lines at the efficiencies for loads which they represent. An important feature of this chart is that the straight-edge can be set to any two given efficiencies for any two loads whatsoever, on which setting the efficiencies at all other loads can be directly read, as can also the percent iron loss and the percent copper loss.

TABLE III

PERCENT EFFICIENCY

	Per Cent. Loss									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	100.00	99.90	99.80	99.70	99.60	99.50	99.40	99.30	99.20	99.10
1	99.01	98.91	98.81	98.71	98.61	98.52	98.42	98.33	98.23	98.13
2	98.04	97.94	97.85	97.75	97.65	97.56	97.46	97.37	97.27	97.18
3	97.09	96.99	96.90	96.80	96.71	96.61	96.52	96.43	96.34	96.24
4	96.15	96.06	95.97	95.87	95.78	95.69	95.60	95.51	95.42	95.32
5	95.23	95.15	95.05	94.96	94.87	94.78	94.69	94.61	94.52	94.43
6	94.34	94.25	94.16	94.07	93.98	93.89	93.80	93.72	93.63	93.54
7	93.46	93.37	93.28	93.19	93.11	93.02	92.93	92.85	92.76	92.68
8	92.59	92.51	92.42	92.33	92.25	92.17	92.08	91.99	91.91	91.83
9	91.74	91.66	91.57	91.49	91.41	91.32	91.24	91.16	91.07	90.99
10	90.91	90.82	90.74	90.66	90.58	90.49	90.41	90.33	90.25	90.17
11	90.09	90.01	89.93	89.84	89.76	89.68	89.60	89.52	89.44	89.36
12	89.28	89.20	89.12	89.04	88.97	88.89	88.81	88.73	88.65	88.57
13	88.49	88.42	88.33	88.26	88.18	88.10	88.03	87.95	87.87	87.79
14	87.71	87.64	87.56	87.49	87.41	87.33	87.26	87.18	87.11	87.03
15	86.95	86.88	86.80	86.73	86.65	86.58	86.50	86.43	86.35	86.28
16	86.21	86.13	86.05	85.98	85.91	85.83	85.76	85.69	85.61	85.54
17	85.47	85.40	85.32	85.25	85.18	85.10	85.03	84.96	84.89	84.82
18	84.74	84.67	84.60	84.53	84.46	84.38	84.31	84.24	84.17	84.10
19	84.03	83.96	83.89	83.82	83.75	83.68	83.61	83.54	83.47	83.40
20	83.33	83.26	83.19	83.12	83.05	82.99	82.92	82.85	82.78	82.71
21	82.64	82.57	82.50	82.44	82.37	82.30	82.24	82.16	82.10	82.03
22	81.97	81.90	81.83	81.76	81.69	81.63	81.56	81.49	81.43	81.37
23	81.30	81.23	81.17	81.10	81.04	80.97	80.90	80.84	80.77	80.71
24	80.64	80.58	80.51	80.45	80.38	80.32	80.25	80.19	80.13	80.06
25	80.00	79.93	79.87	79.80	79.74	79.68	79.62	79.55	79.49	79.43

After all, the labor spent in calculating transformer efficiencies is more or less lost, as the figures have no direct application. Their only value is that they give at a glance some idea of the losses in the transformer, when the losses are not directly given. The same information in a much more usable form is given by the iron and copper losses expressed in watts or in percent of the transformer rating. In calculating the yearly cost of supplying the transformer losses, the actual values of the loss in watts is the data immediately wanted. The fact that a high one-fourth load efficiency means a relatively low iron loss as compared to the copper loss is more evident from a comparison of the relative values of the losses than by noting the relation of the full load to the one-fourth load efficiency.

DETERMINATION OF LOSSES WHEN EFFICIENCIES ARE GIVEN

It is sometimes desired to know the losses of a transformer, when the efficiencies are given. For instance, what is the copper loss of a transformer when the efficiencies are given for full and three-fourths load? Combining equations (80) and (81) to eliminate L_i gives

$$L_c = P\left\{\frac{\eta_{3/4} - (0.75\eta + 0.25\eta\eta_{3/4})}{0.4375\eta\eta_{3/4}}\right\}. \qquad (84)$$

In a similar manner an expression for the copper loss can be obtained when the efficiencies at any other two loads are given.

Example 36: What is the copper loss of a transformer whose full-load efficiency is 0.9752 and whose three-fourths load efficiency is 0.9765? From equation (84),

$$L_c = 5000\left\{\frac{0.9765 - (0.75 \times 0.9752) + (0.25 \times 0.9752 \times 0.9765)}{0.4375 \times 0.9752 \times 0.9765}\right\} = 85 \text{ watts.}$$

This result could have been secured much easier from the chart shown in Fig. 36.

RELATION BETWEEN THE LOSSES FOR THE EFFICIENCIES TO BE A MAXIMUM

If R be the equivalent resistance of a transformer, the total copper loss, ignoring the eddy-current loss in the conductors, is I^2R. Then equation (80) may be written

$$\eta = \frac{EI}{EI + L_i + I^2R}.$$

which may be put into the form

$$\eta = \frac{E}{E + \frac{L_i}{I} + IR}.$$

To obtain the condition for which η is a maximum equate $d\eta/dI$ to zero, which gives

$$L_i = I^2R. \tag{85}$$

This indicates that the efficiency of a transformer will be a maximum when its iron loss is equal to its copper loss. In other words, if the load which is taken from a given transformer be varied by changing the current, other things

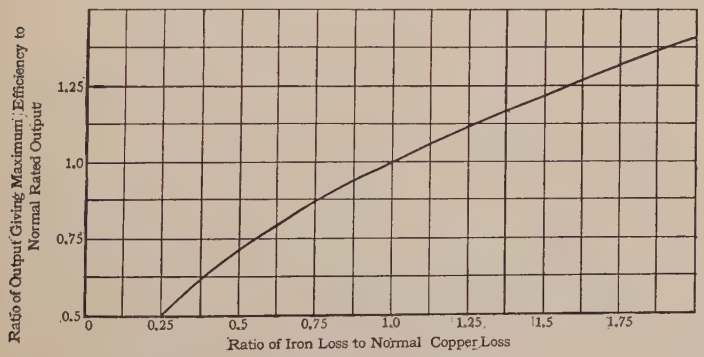


FIG. 37. Variation of the maximum efficiency output with the ratio of iron to copper losses at normal rating.

being constant, the load which will give a maximum efficiency will be the one which produces a copper loss equal to the iron loss.

Example 37: At what percent of normal load must a five k.v.a. transformer operate, which has an iron loss of 42 watts and a copper loss of 85 watts, in order to make its efficiency a maximum?

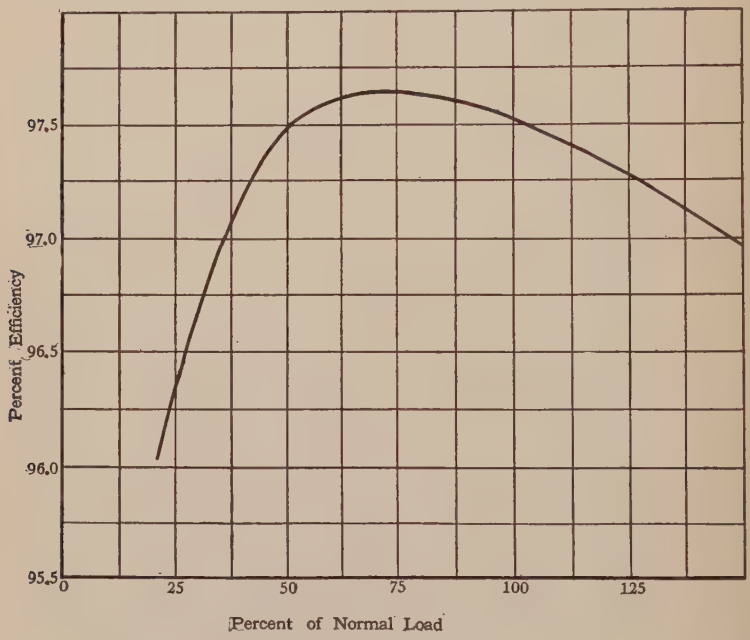


FIG. 38. Efficiency curve of a five k.v.a. transformer.

At maximum efficiency the copper loss must be equal to the iron loss of 42 watts, or 42/85 of its normal value. Since the copper loss varies as the square of the load, the output required to reduce the copper loss to this value is $\sqrt{42/85}$ or practically 70 percent of its normal value. The efficiency curve for this transformer is given in Fig. 38, which shows that the point of maximum efficiency is at approximately 70 percent load.

As a general proposition it may be stated that the ratio of the output that will make the efficiency a maximum to

the normal rated output is equal to the square root of the ratio of the transformer iron loss to the copper loss at normal rated output. Or

$$\frac{\text{Output to give maximum efficiency}}{\text{Normal Output}} = \sqrt{\frac{\text{Iron Loss}}{\text{Copper loss at normal rating}}}. \quad (86)$$

The curve in Fig. 37 gives the maximum efficiency output for various values of the ratio of the iron to the copper loss.

CHAPTER IX

REGULATION

MAGNETIC LEAKAGE has already been referred to as being the cause of an eddy-current loss in the conductors, but no reference was made to another phase of this leakage, which is the introduction of an element affecting the regulation of the transformer. The number of leakage lines flowing in a given transformer is largely dependent upon the relative position of the primary and secondary windings. Assuming, for instance, a transformer whose primary and secondary windings could occupy the same space, the primary winding would tend to establish a leakage field in one direction and the secondary to establish a leakage field in the opposite direction, resulting in the non-existence of an actual leakage field. If now the primary and secondary coils be separated, the forces tending to establish fields around the coils, no longer being coincident and no longer completely annulling each other, set up actual leakage fields. Thus if the coils were sufficiently separated, none of the lines of force from the primary coil will reach the secondary, in which case all of the lines through the primary winding will be leakage lines. The leakage fluxes from the two coils produce an effect which is additive and it is allowable, for the sake of simplicity, to think of all of the leakage lines as encircling the primary winding instead of being divided between the two parts. The equivalent leakage flux through the primary coil produces a counter electromotive force, which absorbs part of the impressed electromotive force. It is evident that this part of the impressed electromotive force does not reach the secondary winding and, therefore, its magnitude is a measure of the magnetic separation of the

windings. The counter e.m.f. due to the leakage flux is at right angles to the counter e.m.f. due to the normal flux in the magnetic circuit at no load. This, of course, assumes that the load current is in phase with the terminal voltage; that is, that the transformer is delivering current to a non-inductive load. The phase relation of the counter e.m.f. due to the normal flux in the magnetic circuit at no load becomes evident when it is considered that the leakage flux is in time phase with the primary load current, which in turn is displaced 90 degrees in time phase relation from the normal flux in the magnetic circuit at no load.

If the secondary of a transformer, the resistance of whose windings is negligible, be short-circuited, the voltage delivered to the load becomes zero and the voltage impressed on the primary winding is only that required to balance the counter e.m.f. due to the leakage flux. The voltage in this case, when normal full-load current flows through the short-circuited secondary, becomes the measure of the magnetic separation of the windings. Of course a transformer winding has resistance and in this test the impressed voltage is partly used in sending full-load current through this resistance. Considering the short-circuited transformer as an inductive coil, the impressed voltage required to drive full-load current through its windings is its impedance voltage. This impedance drop may be resolved into two components at right angles to each other, the one in time phase relation with the current being the resistance drop through the windings and the second being the reactive element which results from the magnetic leakage. The percent impedance of the transformer is the impedance voltage expressed as a percentage of the normal voltage of the winding upon which the impedance volts drop was determined. The percent reactance of a transformer is the reactive volts drop expressed as a percentage in a similar manner. The percent reactance cannot be measured directly but must be calculated

from the percent impedance drop, the percent drop in phase with the current and the known relation between these three quantities. The percent voltage drop in phase with the current has the same value as the total copper loss, expressed as a percentage of the k.v.a. output rating of the transformer.

CALCULATION OF TRANSFORMER REACTANCE

It is sometimes necessary to know the percent reactive drop, when the impedance value is not known. The following method of approximating this reactance, from the constants of the transformer, is only approximate, as certain assumptions must be made as to the length and section of the leakage paths which are not exact. The method applies particularly to the concentric type of winding shown in Fig.

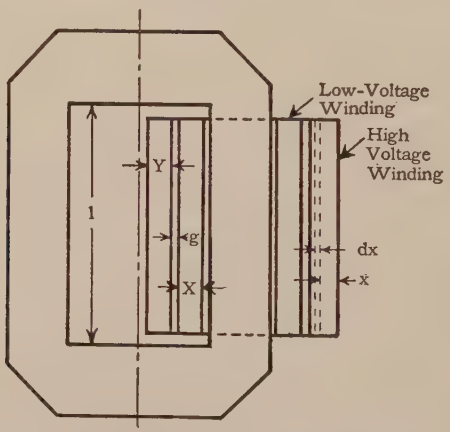


Fig. 39. Section through a concentric winding of a core-type transformer.

39, but the same method can be applied to the interleaved type of winding.

Since the currents in the high- and low-voltage windings of a transformer are opposite in time phase relation, at

each point along the gap separating them there is a magnetomotive force equal to the ampere-turns in either winding tending to produce a magnetic flux through the gap. This m.m.f. is $4\pi/10\ IT$, IT being the ampere-turns in either winding. The reluctance of the leakage path is $l/l_g g$, l being the length of the leakage path, l_g the mean turn of the gap between windings and g the width of the gap. The flux through the gap is

$$\Phi_g = 2.54 \frac{4\pi}{10} \frac{IT l_g g}{l}. \tag{87}$$

The constant 2.54 permits the linear distances to be expressed in inches instead of centimeters. This flux is in phase with the current in the winding and is therefore 90 degrees in time phase relation from the flux producing the terminal voltage. As stated before, the leakage flux may be thought of as giving rise to an effect equivalent to an inductance connected in series with the transformer. By definition the inductance L of a coil, in c.g.s. units, is equal to the flux turns which will be produced through the coil by unit current. Therefore, when L is expessed in henrys,

$$L = \frac{10\phi T}{10^9 I}.$$

Substituting the value of Φ in this expression from equation (87) gives

$$L = \frac{32.0}{10^9} \frac{T^2}{l} l_g g.$$

This gives a reactance voltage of $e = 2\pi f L I$ or

$$e = \frac{200}{10^9} \frac{f T^2 I}{l} l_g g.$$

Expressing this as a percentage of the voltage E of the

transformer gives

$$\text{Percent reactance for flux through air-gap} = \frac{2}{10^5}\frac{fT^2I}{lE}l_g g. \qquad (88)$$

The m.m.f. tending to produce leakage through the windings is a maximum at the inner edges adjacent to the gap between them and decreases to zero at the outer edges. Between these limits, it is proportional to the distance from the gap. As in the discussion of eddy-current loss, it is assumed that there are two main paths of the leakage flux in the windings, one returning through the magnetic circuit and one being entirely within the coil itself. Consider first the leakage path which returns through the magnetic circuit of the transformer. The m.m.f. acting on the small section of the primary windings of width dx, shown in Fig. 39, is $(4\pi x/10Y)IT$. The reluctance of this leakage path is $l/l_p dx$, where l_p is the mean turn of the primary winding. Then

$$d\Phi_p = 2.54\frac{4\pi}{10}\frac{x}{Y}IT\frac{l_p}{l}dx.$$

This flux encircles the fractional part x/Y of the turns of the coil and therefore the fractional part x/Y of the flux should be counted as encircling the entire coil, so that

$$d\Phi_p = 2.54\frac{4\pi}{10}\frac{x^2}{Y^2}IT\frac{l_p}{l}dx$$

or

$$\phi_p = 2.54\frac{4\pi}{10}IT\frac{l_p}{l}\frac{1}{Y^2}\int_0^Y x^2dx$$

or

$$\Phi_p = 2.54\frac{4\pi}{10}(IT)_p\frac{l_p}{l}\frac{Y}{3}.$$

It has been assumed that the mean turn of the sectional area of width dx has a constant length equal to the mean

turn of the coil. This assumption will not introduce an error greater than that to which the final result is subject, due to the uncertainty regarding the length of the leakage path. Similarly the leakage flux through the secondary coil is

$$\Phi_s = 2.54 \frac{4\pi}{10} (IT)_s \frac{l_s}{l} \frac{X}{3}.$$

The percent reactance due to this leakage flux through the windings may be determined in the same manner as used for deriving equation (88) as follows:

$$\begin{matrix}\text{Percent reactance} \\ \text{for flux through primary}\end{matrix} = \frac{2}{10^5} \frac{f(IT^2)_p}{lE_p} \frac{l_p Y}{3}, \qquad (89)$$

$$\begin{matrix}\text{Percent reactance} \\ \text{for flux through secondary}\end{matrix} = \frac{2}{10^5} \frac{f(IT^2)_s}{lE_s} \frac{l_s X}{3}. \qquad (90)$$

Since $(IT^2)_p/E_p = (IT^2)_s/E_s$ equation (90) may be written

$$\begin{matrix}\text{Percent reactance} \\ \text{for flux through seconadry}\end{matrix} = \frac{2}{10^5} \frac{f(IT^2)_p}{lE_p} \frac{l_s X}{3}. \qquad (91)$$

Adding equations (88), (89) and (91) gives the total percent reactance of the transformer due to the leakage paths which return through the magnetic circuit as

$$\begin{matrix}\text{Total percent reactance} \\ \text{for flux returning} \\ \text{through magnetic circuit}\end{matrix} = \frac{0.66}{10^5} \frac{fIT^2}{lE} (3l_g g + l_p Y + l_s X). \qquad (92)$$

It will be noted that $l_s X$, $l_p Y$ and $l_g g$ are the areas of the secondary winding, primary winding and duct respectively, at right angles to the path of the leakage flux. This will suffice to outline the procedure in case the windings or ducts are not uniform in thickness.

In addition to this leakage flux, whose path is partly in air and partly in the magnetic circuit, there are additional

leakage lines in the coils themselves whose paths do not enter the magnetic circuit. The effect of such lines is much less than that of the leakage approximated by equations (89) and (90). The reactance due to the leakage in the coils themselves can be approximately by the same method as used for the main leakage flux. Making the same assumptions as when calculating the eddy-current loss due to leakage flux through the windings, the flux through a small path dx in the coil is

$$d\Phi_p = 2.54 \frac{4\pi}{10} \frac{l_p}{lY} IT \frac{K^2}{1 + K^2} x dx.$$

Since this flux encircles the fractional part $4Kx^2/lX$ of the total turns, this fractional part of the total flux should be counted as encircling the entire coil, therefore,

$$d\Phi_p = 2.54 \frac{4\pi}{10} \left(\frac{4K^3}{1 + K^2} \right) \frac{l_p}{l^2 Y^2} ITx^3 dx$$

or

$$\Phi_p = 2.54 \frac{4\pi}{10} \left(\frac{4K^3}{1 + K^2} \right) \frac{l_p}{l^2 Y^2} IT \int_0^{Y/2} x^3 dx$$

or

$$\Phi_p = 0.199 \left(\frac{K^3}{1 + K^2} \right) \frac{l_p Y^2}{l^2} IT.$$

For the values of K ordinarily existing in transformers, $K^3/(1 + K^2)$ may be written as K, then

$$\Phi_p = 0.199 IT \frac{l_p}{l} Y.$$

Following the same procedure as before,

$$\text{Percent reactance for flux within secondary} = \frac{0.125}{10^5} f \frac{IT^2}{lE} l_p Y,$$

$$\text{Total percent reactance for flux within windings} = \frac{0.125}{10^5} f \frac{IT^2}{lE} (l_p Y + l_s X).$$

The total reactance is therefore

$$\text{Total percent reactance} = \frac{fIT^2}{10^5 lE}[2l_g g + 0.79(l_p Y + l_s X)]. \quad (93)$$

From the above it is evident that the part of the reactance due to the leakage flux through the coils and returning in the magnetic circuit is more than five times as large as the part due to the leakage flux within the windings. In order to obtain the total reactance of a transformer winding, the reactance of each high-low group must be calculated separately and the results added. A transformer winding may be composed of a number of *H-L* groups; for example, if the winding is composed of a high-voltage coil between

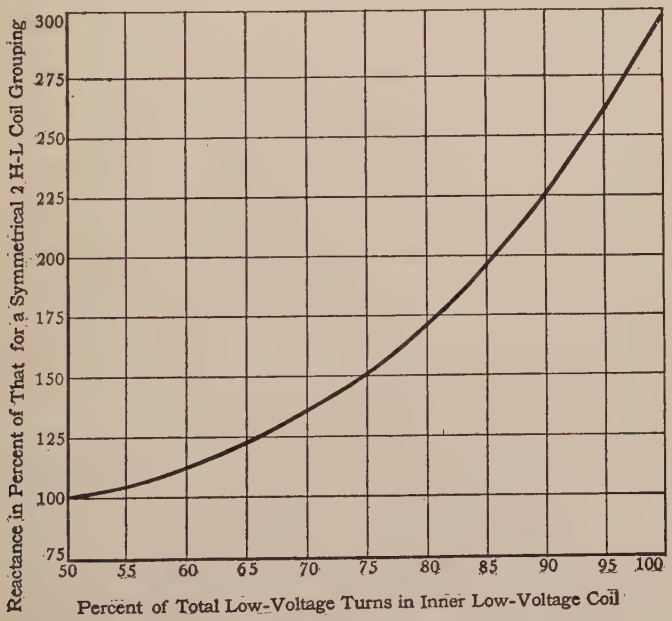


FIG. 40. Effect on the reactance of a concentric transformer winding produced by changing from a symmetrical to an unsymmetrical coil grouping. The total thickness of the low-voltage winding remains constant and equal to the thickness of the high-voltage winding.

two low-voltage coils, the winding is said to have a 2 *H-L* coil grouping, the number of *H-L* groups being the number of times the high- and low-voltage coils are adjacent to each other. Increasing the number of *H-L* groups decreases the magnetic leakage and thus reduces the eddy-current loss and the transformer reactance. This is largely due to the ampere turns producing leakage flux being distributed and not concentrated as with a single *H-L* grouping. The curve in Fig. 40 gives an idea as to the variation of the reactance of a given transformer, with change of the grouping. It is assumed that the total coil thickness remains constant, as well as the relative thickness of the high- and low-voltage coils.

Example 38: What is the percent reactance of a 50 k.v.a., 2,300 to 230 volt, 60-cycle transformer, having the following constants? This transformer has a 2 *H-L* winding group, but for the sake of simplicity we will calculate the reactance for one of the groups only, taking the mean turn of the air-gap as the average of the two gaps.

$f = 60$,	$l_s = 26$,	$Y = 1$,
$T = 200$,	$E = 2300$,	$l_g = 30$,
$\iota = 21.8$,	$X = 1$,	$g = 0.2$.
$l = 10$,	$l_p = 34$,	

$$\text{Percent reactance} = \frac{60 \times 21.8 \times 200^2}{10^5 \times 10 \times 2300}(2 \times 30 \times 0.2 + 0.79(34 \times 1 + 26 \times 1)) = 1.36.$$

Assuming that the reactance of both *H-L* groups is the same, the total reactance would be 2.72 percent.

TRANSFORMER REGULATION

The regulation of a transformer is defined as the drop in secondary voltage, from no-load to full load, in terms of the full-load voltage. In the calculation of transformer regulation, there are three different conditions of load which must be considered. These are:

1. With load of unity power-factor.
2. With load of lagging power-factor.
3. With load of leading power-factor.

With a load of unity power-factor, the relation of the various factors involved is shown in Fig. 41, where E_1 is the secondary voltage at no load and E_2 the secondary voltage at full load. The quantities IR, IX and IZ are the resistance, reactance and impedance drops in the transformer at normal load in terms of its characteristics as referred to the secondary side. By definition,

$$\text{Regulation} = \frac{E_1 - E_2}{E_2} = \frac{AF}{E_2} = \frac{AC + CF}{E_2}. \qquad (94)$$

In equation (8), AC is the IR volts drop and CF is

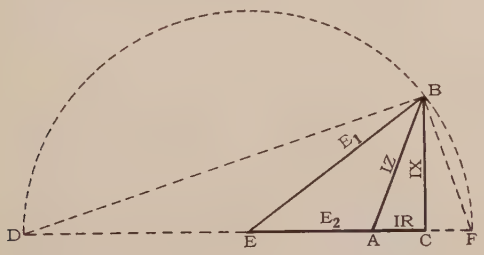


FIG. 41. Regulation diagram for load of unity power-factor.

determined as follows, from the similar triangles DBC and BCF, Fig. 41,

$$\frac{DC}{BC} = \frac{BC}{CF}$$

or

$$CF = \frac{(BC)^2}{DC}. \qquad (95)$$

From Fig. 41 it is apparent that DC is approximately equal to $2E_2$, since the relative values of AC and E_2 are something like 5 to 100. Therefore, since BC is equal to IX,

$$CF = \frac{(IX)^2}{2E_2}.$$

Equation (94) may now be put into the form

$$\text{Regulation} = \frac{IR + \dfrac{(IX)^2}{2E_2}}{E_2}$$

or

$$\text{Percent Regulation} = \text{Percent } IR + \frac{(\text{Percent IX})^2}{200}. \quad (96)$$

With a load having a lagging power-factor, the various

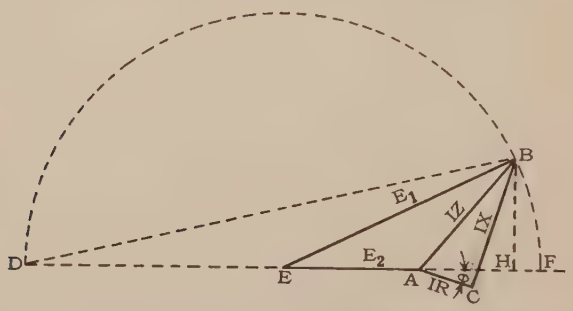


FIG. 42. Regulation diagram for load of lagging power-factor.

quantities have the relation shown in Fig. 42. By definition,

$$\text{Regulation} = \frac{E_1 - E_2}{E_2} = \frac{AF}{E_2} = \frac{IR\cos\theta + IX\sin\theta + HF}{E_2}.$$

From Fig. 42

$$HF = \frac{(BH)^2}{2E_2}.$$

This is a relation similar to the one shown by equation (95). But

$$BH = IX\cos\theta - IR\sin\theta$$

or

$$\text{Regulation} = \frac{IR\cos\theta + IX\sin\theta}{E_2} + 0.5\frac{(IX\cos\theta - IR\sin\theta)^2}{E_2},$$

$$\text{Percent Regulation} = \text{Percent } IR\cos\theta + \text{Percent } IX\sin\theta + \frac{(\text{Percent } IX\cos\theta - \text{Percent } IR\sin\theta)^2}{200}.$$

For ordinary use the last term of this expression may be omitted. Then

$$\text{Percent Reg.} = \text{Percent } IR\cos\theta + \text{Percent IX}\sin\theta. \quad (97)$$

With a load having a leading power-factor, the conditions

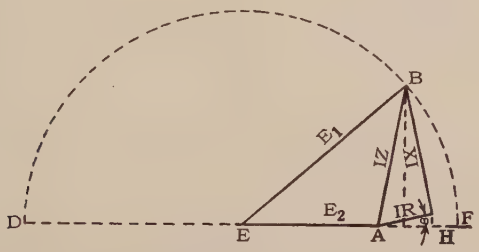


FIG. 43. Regulation diagram for load of leading power-factor.

are shown by Fig. 43. By definition,

$$\text{Regulation} = \frac{E_1 - E_2}{E_2} = \frac{AF}{E_2} = \frac{IR\cos\theta - IX\sin\theta + HF}{2E_2}.$$

From Fig. 43,

$$HF = \frac{(BH)^2}{2E_2}.$$

This is a relation similar to the one shown by equation (95). But

$$BH = IX\cos\theta + IR\sin\theta$$

Hence,

$$\text{Regulation} = \frac{IR\cos\theta - IX\sin\theta}{E_2} + 0.5\frac{(IX\cos\theta + IR\sin\theta)^2}{E_2}$$

or

$$\text{Percent Regulation} = \text{Percent } IR \cos\theta - \text{Percent } IX \sin\theta + \frac{(\text{Percent } IX \cos\theta + \text{Percent } IR \sin\theta)^2}{200}.$$

For ordinary use the last term of this expression may be omitted without error, then

$$\text{Percent Reg.} = \text{Percent } IR \cos\theta - \text{Percent } IX \sin\theta. \quad (98)$$

Example 39: What is the percent regulation with a 100 percent power-factor load of a five k.v.a. transformer which has a copper loss of 85 watts and an impedance of three percent?

If the transformer has an impedance of three percent and a resistance drop of 1.7 percent, the reactance drop is $3^2 - 1.7^2$ or 2.47 percent. Then, the regulation is, from equation (96),

$$\text{Percent Regulation} = 1.7 + \frac{(2.47)^2}{200} = 1.73.$$

The regulation of the same transformer with an 80 percent lagging power-factor load is, from equation (97),

Percent Regulation $= 1.7 \times 0.8 + 2.47 \times 0.6 = 2.84$.

The regulation with a load having a leading power factor of 90 percent is, from equation (98),

Percent Regulation $= 1.7 \times 0.9 - 2.47 \times 0.44 = 0.44$.

With a load having a leading power-factor load of 80 percent, the regulation would be negative. That is, the voltage delivered at full load would be greater than the no-load voltage.

CHAPTER X

HEATING

In transformer operation, the amount of temperature rise is important because of the necessity of keeping the temperature of the windings from exceeding a safe value. While the temperature of the magnetic circuit is not important, since the use of silicon steel renders it non-aging even at relatively high temperatures, yet its temperature must be limited because of its proximity of the windings.

A temperature rise curve for a transformer winding may be divided into three parts, as shown in Fig. 44. From the

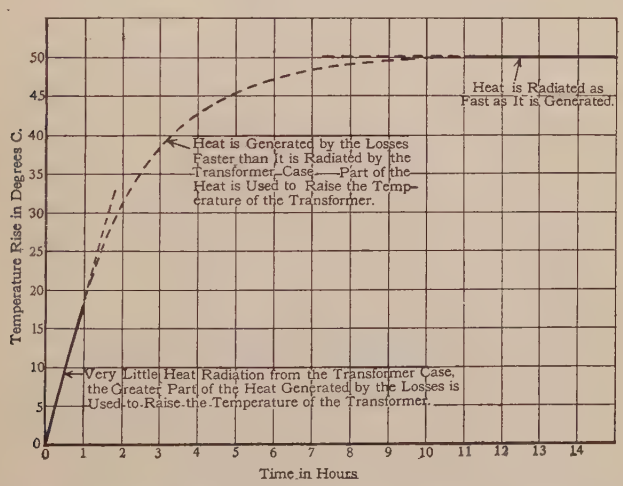

Temperature Rise in Degrees C.
Time in Hours
Heat is Radiated as Fast as It is Generated.
Heat is Generated by the Losses Faster than It is Radiated by the Transformer Case — Part of the Heat is Used to Raise the Temperature of the Transformer.
Very Little Heat Radiation from the Transformer Case, the Greater Part of the Heat Generated by the Losses is Used to Raise the Temperature of the Transformer.

Fig. 44. Temperature rise of a transformer winding, showing the three parts into which the curve may be divided.

origin, the first part of the curve is practically a straight line, which means that when a cold transformer is put into

service, the rate of temperature rise is nearly constant for a time. This condition signifies that the greater part of the heat from the losses is absorbed in raising the temperature of the transformer, and that very little heat is radiated into the air. The rate of temperature rise is represented by the slope of the line, and depends only on the amount and kind of material in the transformer and on the total amount of heat generated by the losses.

The second part of the curve represents the condition when the initial rate of temperature rise is not maintained, because of the heat beginning to flow away from its source of generation. After the heat commences to be radiated from the surface of the transformer case into the surrounding air, the rate of rise of temperature falls very rapidly. The temperature rise of the transformer continues to increase at a decreasing rate, until the heat is radiated at the same rate as it is being generated by the losses. When this condition, represented by the third part of the curve in Fig. 44, is reached, the temperature rise curve has flattened out showing that the rate of increase of temperature is practically zero, and the maximum temperature rise has been obtained.

INITIAL RATE OF TEMPERATURE RISE

When putting a cold transformer into service, the initial rate of temperature rise of the windings depends only on the copper loss, the amount of copper in the coils and the specific heat of the copper. This condition exists only for a short time until the heat from the windings begins to be imparted to the oil and the other parts of the transformer. The specific heat of copper is 0.092, or 0.092 gram-calories of heat are required to raise the temperature of one gram of copper one degree C. Putting this value of the specific heat in more convenient terms, it is found that approximately 2.9 watt-minutes are required to raise the temperature of one pound of copper one degree C.

In calculating the initial rate and the actual temperature rise after a given time, before the rise is arrested by conduction, convection or radiation of heat, this relation may be expressed in the following form:

$$\frac{t}{T} = \frac{W_c}{2.9}. \tag{99}$$

where t is the rise in temperature expressed in degrees centigrade in T minutes, or t/T is the initial rate of temperature rise and W_c is the watts copper loss per pound of copper in the winding.

Example 40: What is the initial rate of temperature rise in a transformer winding when the copper is operating at seven watts per pound? From equation 99,

$$\frac{t}{T} = \frac{7}{2.9} = 2.42 \text{ degrees C. per minute.}$$

Example 41: At what value of watts per pound of copper is it permissible to work a transformer which is used for starting a motor when the time required for starting is one minute, and the permissible temperature rise is 100 degrees C.? From equation 99,

$$W_c = \frac{100}{1} \times 2.9 = 290 \text{ watts per pound.}$$

FINAL TEMPERATURE RISE OF THE OIL

The first step in predetermining the temperature rise of the windings of a transformer, after the straight line part of the temperature rise curve has been passed, is to determine the temperature rise of the oil. The rate P at which a hot body emits heat is very nearly proportional to the excess t of its temperature above that of its surroundings, and roughly proportional to the area a of its surface, that is, $P = aet$ in which e is the proportionaly factor, which is called the emissivity of the surface of the body. Solving this equation for t gives

$$t = \frac{P}{ea}. \tag{100}$$

The value of e for various types of transformer cases when P is expressed in watts, a in square inches and t in degrees C. is as follows:[1]

For smooth cast iron cases ..... $e = 0.0075$
For smooth boiler iron cases ..... $e = 0.007$
For corrugated cast iron cases ..... $e = 0.0059$
For corrugated sheet iron cases ..... $e = 0.0043$
$4\frac{5}{32}$ inch pitch and $3\frac{1}{2}$ inch depth of corrugation

These constants are based on approximately 40 degrees C. rise of the oil and the radiating surface calculated from the hot oil level in the case at approximtely 80 degrees C.

Example 42: What will be the temperature rise of the oil in a 50 k.v.a. transformer, which has a total loss of 850 watts and a corrugated cast iron surface of 3,600 square inches? From equation 100,

$$T = \frac{850}{0.0059 \times 3600} = 40 \text{ degrees C.}$$

TEMPERATURE GRADIENT BETWEEN WINDINGS AND OIL

The difference between the temperature rise of the transformer winding, as determined by the increase of resistance method and the temperature rise of the oil, is the temperature gradient between the windings and the oil. The temperature gradient, as thus determined, is less than the maximum gradient which might be determined by a thermocouple placed in the hottest spot in the windings. It is generally assumed that the temperature of the hottest spot is never greater than about 10 degrees C. above the average temperature indicated by the increase of resistance measurement. The measured temperature gradient as indicated by the increase of resistance method is usually between five and fifteen degrees C. The value of this gradient in a particular transformer depends upon the area of the winding exposed to the oil and the thickness of the coils. The nature of the coil insulation will also affect the value of the gradient, as a high-voltage winding having a relatively large amount

[1] See *Proc. A. I. E. E.*, Vol. XXX, No. 3, page 463.

of insulation will be likely to have a high gradient, other things being equal. The viscosity of the cooling oil will also have a slight effect on the gradient.

In a particular transformer it is evident that the gradient will be proportional to the total loss in the winding, and will therefore vary as the square of the load on the transformer. For example, a transformer which has a ten degrees C. gradient at 100 percent load will have approximately 15.5 degrees C. gradient at 125 percent load.

It is possible to predetermine the temperature gradient in a coil approximately by assuming that the total drop in temperature is that used to force the heat through the insulation and copper in the coil. The rate of flow of heat through a conducting body may be expressed by the relation

$$P = \lambda \frac{a}{i} t. \tag{101}$$

where λ represents the heat conductivity of the material, a the area of the section through which the heat is flowing expressed in square inches, i the thickness of the material and t the resulting temperature drop or temperature gradient. Equation (101) may be put into the form

$$t = \frac{1}{\lambda} \frac{i}{a} P = \frac{1}{\lambda} W_a i. \tag{102}$$

where W_a is the watts per square inch being forced through the section of area a. The approximate value of λ for fibrous insulating material expressed as the number of gram calories of heat per second being forced through a cubic centimeter of the material with a drop of temperature of one degree centigrade, is 0.00032. Expressing this value in the more convenient form of watts being forced through a cubic inch, with a drop of temperature of one degree centigrade, for fibrous insulation $\lambda = 0.0053$ approximately.

Example 43: What is the temperature gradient through a sheet of insulating material 1/16 inch thick, through which heat is being forced at the rate of one watt per square inch? From equation (102),

$$t = \frac{1}{0.0053} \times 1.0 \times 0.062 = 11.6 \text{ degrees C.}$$

This indicates that a solid sheet of insulation close up against a coil cuts off the flow of heat across the area covered to a considerable extent. To get the total gradient in the coil in this case, the drop in temperature within the coil would have to be added to that through the external insulating sheet. Each coil in a transformer winding should have, if possible, some surface exposed to the cooling oil, unless some other condition exists which permits the heat to escape. Coils wound with relatively heavy conductors will have a considerable amount of heat conducted out through their leads. A section through a coil which is adjacent to an insulating sheet is shown in Fig. 45. The heat is generated uniformly throughout the coil, and since one side is insulated the greater part of the heat must pass out through the free surface. This condition is therefore different from that given in the example above, where the heat is forced through the insulating sheet by an outside source and the sheet has no loss generated within itself. The drop in temperature across the small section of the coil of length dx in Fig. 45, by equation 102, is

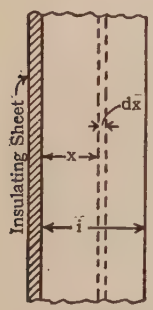


FIG. 45. Section of a transformer coil adjacent to a sheet of insulating material.

$$dt = \frac{1}{\lambda} W_a dx.$$

The total loss in the part of the coil between the insulating sheet and the small section dx is $(x/i)W_a$. In this case

W_a is the watts per square inch of the coil surface exposed to the cooling action of the oil. Then,

$$dt = \frac{1}{\lambda}\frac{W_a}{i}xdx$$

or

$$t_{\max.} = \frac{1}{\lambda}\frac{W_a}{i}\int_o^i xdx = \frac{1}{\lambda}\frac{W_a}{2}i. \qquad (103)$$

This is the gradient from a point adjacent to the insulating sheet across the entire coil, and is therefore greater than the difference between the point of average temperature and the temperature of the oil. Assuming that the average temperature along this path of heat flow is at a distance of 0.5 i from the insulating sheet

$$t_{av.} = \frac{1}{\lambda}\frac{W_a}{i}\int_{i/2}^i xdx = \frac{1}{\lambda}\frac{W_a}{2.7}i. \qquad (104)$$

A comparison of these two equations indicates that the average gradient, or the value which would be given by an increase of resistance measurement will be something like 75 percent of the maximum gradient. Equations 103 and 104 assume that the insulating material is distributed uniformly throughout the coil, and i is the total thickness of this insulation, or is the total thickness of the coil exclusive of the copper. Ordinarily the temperature drop through the copper may be ignored because of the shortness of the heat path through the copper, and because of the relatively great heat conductivity of copper. The heat conductivity of copper is 8.5 as compared to 0.0053 for fibrous insulating material, so that the temperature drops through paths of heat flow of the same dimensions would be in the ratio of 1 to 2,500 for the two materials. If both sides of the coil are exposed to the cooling oil, the heat can be conducted in both directions, and the value of i in equations

(103) and (104) should be taken as one-half of the total thickness of insulation in the coil.

Example 44: What is the average temperature drop along the path of heat flow shown by the dotted line in Fig. 46, when the value of Wa is 0.8 and the total thickness of the insulation along this path is 0.2 inches? From equation (104),

$$t_{av.} = \frac{1}{0.0053} \times \frac{1.0}{2.7} \times 0.2 = 14 \text{ degrees C.}$$

It must not be taken for granted that this method of calculating temperature gradients, with the constants given, will always give accurate results. The value of λ varies for the different insulating materials, and the effect of the impregnation treatment on the different methods of insulation is another variable. The general conditions vary with different coil arrangements, and with different types of transformers. In the simplest case there are a number of heat flow paths in series, in parallel or in series-parallel, of which only the most important ones can be considered in a particular calculation. While ordinarily the drop in temperature along the copper is negligible, there are certain cases similar to that along the dotted line B in Fig. 46, where the gradient is appreciable under certain conditions.

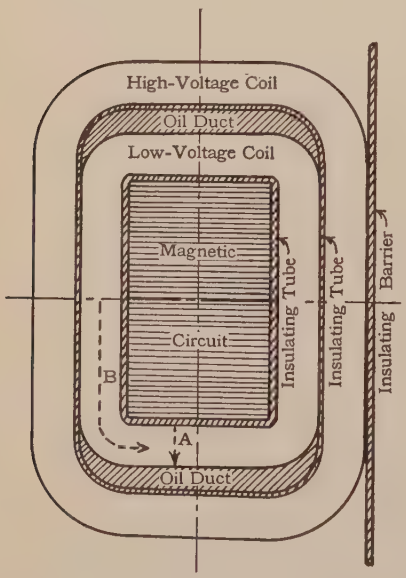


Fig. 46. Section of one leg of a core-type transformer with concentric coils.

This gradient can be calculated with some degree of certainty because the heat conductivity of copper is a fixed quantity. For this case it is more convenient to have equation 104 in terms of the watts per pound of copper loss in the coil W_c, than in terms of W_a. Since $W_a = 0.321\ W_{ci}$, equation 104 may be written, substituting the value of 8.5 for the heat conductivity of copper,

$$t_{av.} = 0.014\ W_{ci}^2. \qquad (105)$$

Example 45: What is the temperature gradient along the length of the conductors corresponding to the dotted line *B* in Fig. 46, in a transformer coil, when the length of the path is approximately five inches, and the copper is working at five watts per pound? From equation (105),

$$t_{av.} = 0.014 \times 5 \times 5^2 = 1.75 \text{ degrees C.}$$

HEATING CURVE

Assume that the transformer is composed of a mass of material weighing G pounds which has a specific heat of S, expressed as the number of watts required to raise the temperature of one pound of the material one degree C. per minute. At some point on the heating curve the small increase dt in the temperature rise occurs during a small interval of time dT. The rate of increase of temperature rise is

$$\frac{dt}{dT} = \frac{L - P}{GS}.$$

where L is the total watts loss in the transformer and P the watts being radiated into the air with the transformer at temperature t. The watts L-P is the rate at which heat is being absorbed in raising the temperature of the transformer, and this divided by GS, or the watt minutes required to raise the temperature of the transformer one degree, gives the rate of temperature rise increase at this point on the heating curve. Substituting the value of P from equation

100 gives

$$\frac{dt}{dT} = \frac{L - aet}{GS}.$$

When the final temperature rise has been reached or the heating curve has flattened out,

$$L = P = aet_r,$$

where t_r is the final temperature rise of the transformer, then,

$$\frac{dt}{dT} = \frac{ea(t_r - t)}{GS}$$

or

$$-\frac{dt}{t_r - t} = -\frac{ea}{GS}dT.$$

The integral of this expression is

$$\log_\epsilon (t_r - t) + \text{Constant} = -\frac{ea}{GS}T.$$

Since, when T is equal to zero t is also equal to zero,

$$\text{Constant} = -\log_\epsilon t_r$$

Therefore,

$$\log_\epsilon (t_r - t) - \log_\epsilon t_r = -\frac{ea}{GS}T.$$

$$\frac{t_r - t}{t_r} = \epsilon^{-(ea/GS)T}$$

or

$$t = t_r (1 - \epsilon^{-(ea/GS)T}). \qquad (106)$$

This expression may be put into the form

$$t = t_r(1 - \epsilon^{-(T/T_0)}). \qquad (107)$$

where

$$T_0 = \frac{GS}{ea} \quad \text{or} \quad = \frac{GSt_r}{eat_r}$$

$$= \frac{\text{heat required to reach final temperature}}{\text{rate of generation of heat}}. \quad (108)$$

where T_0 = the time required to reach final temperature if there is no heat radiation to the surrounding air. When T is equal to T_0, that is, when T/T_0 is equal to unity, equation (107) indicates that the temperature rise would be approximately 63 percent of the final rise, or in other words in time T_0 the transformer would reach 63 percent of the temperature it would attain if there was no radiation of heat.

Equation 107 will apply either to the temperature rise of the oil or of the windings. When the rise of the oil is being considered, t_r refers to the final rise of the oil, and for the windings t_r represents the final rise of the oil plus the final value of the temperature gradient between the oil and windings.

No allowance is made for the increase in the copper loss of the transformer as its temperature rises. If the value of the copper loss used in calculating the data for a heating curve is approximately the loss at the temperature at which the heating curve becomes flat, each point on the curve will be slightly higher than the actual temperature of the transformer. This error is just opposite to another introduced by the changing viscosity of the oil as the transformer heats.

The values for the specific heat of copper, iron and oil, expressed in watt minutes per pound per degree centigrade are approximately as follows:

$S_c = 2.9$ for copper,
$S_i = 3.6$ for iron,
$S_o = 1.5$ for oil.

Example 46: What is the time constant T_0 for a transformer when $G_c = 45$ lbs., $G_i = 188$ lbs., $G_o = 34$ lbs., $e = 0.0059$ and $a = 700$ square inches?

$$G_cS_c + G_iS_i + G_oS_o = 45 \times 2.9 + 188 \times 3.6 + 34 \times 1.5 = 862.$$

From equation (108),

$$T_0 = \frac{862}{0.0059 \times 700} = 208 \text{ minutes} = 3.46 \text{ hours.}$$

Example 47: Draw the curves for the temperature rise of the oil and the windings of the transformer whose constants are given in the preceding example, whose final temperature gradient is 12 degrees Centigrade, and the sum of whose losses at 75° C. is 180 watts.

From equation (107),

$$t_r \text{ for the oil} = \frac{180}{0.0059 \times 700} = 43 \text{ degrees C.}$$

$$t_r \text{ for the windings} = 43 + 12 = 55 \text{ degrees C.}$$

Then for the oil, from equation (107),

$$t = 43(1 - \epsilon^{-(T/208)})$$

which is a relation between time T and temperature rise t, for this particular transformer. The curve is shown in Fig. 47 for both the oil and the windings.

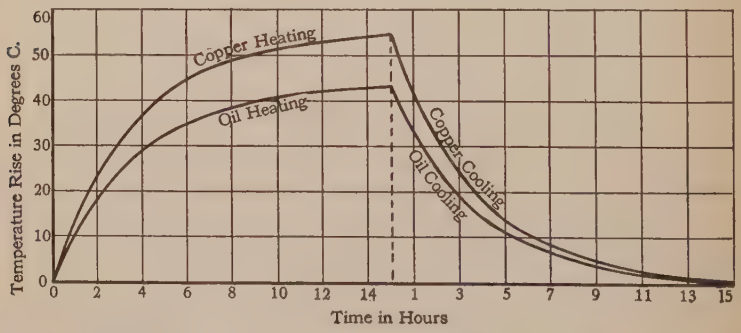


FIG. 47. Relation of heating and cooling for the oil and copper in a transformer obtained by the use of equations (107) and (109).

For convenience in plotting similar curves, values of $(1 - \epsilon^{-(T/T_0)})$ for various values of T/T_0 are given in the

form of a curve in Fig. 48, which indicates that on a heating curve a transformer will reach 98 percent of its final temperature, or practically a constant temperature for a value of T/T_0 of 4.0. If the time constant of a particular trans-

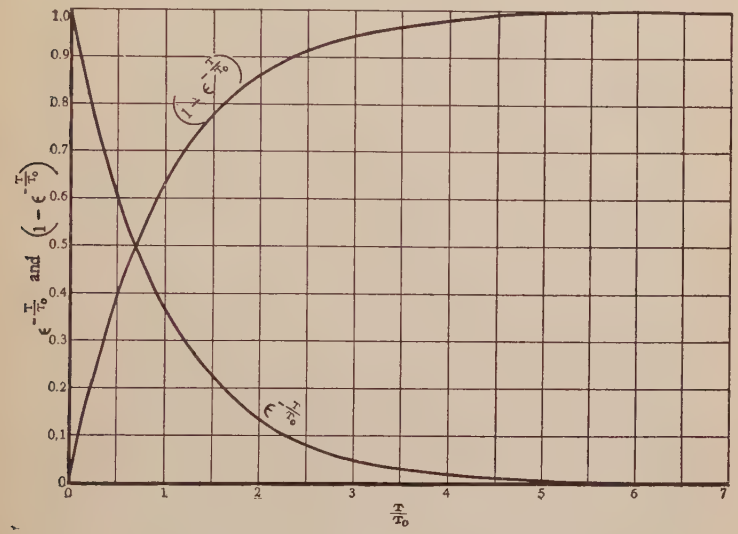


FIG. 48. RELATION OF $\left(1-\epsilon^{-\frac{T}{T_0}}\right)$ AND $\epsilon^{-\frac{T_0}{T_0}}$

For use in estimating the temperature rise of transformers under various service conditions.

former is three hours or less, its final temperature will be reached in not more than 12 hours. If its time constant is more than three hours, it will require more than 12 hours to reach a practically steady temperature.

COOLING CURVE

The cooling curve of a transformer is sometimes important in determining its suitability to take care of special service conditions. The equation for the cooling curve may be determined by the same method as used for the heating curve.

The amount of heat radiated by the transformer during a small interval of time dT is

$$dPdt = aetdT.$$

From equation 100, let dt be the small reduction in temperature of the transformer during this period of time dT. The amount of heat which must be taken from the transformer to reduce its temperature dt degrees C. is $GSdt$. Therefore,

$$aetdT = GSdt$$

or

$$-\frac{dt}{t} = -\frac{ea}{GS}dT.$$

The integral of this expression is

$$\log_\epsilon t + \text{Constant} = -\frac{ea}{GS}T.$$

When T is equal to zero, t is equal to t_r, therefore,

$$\text{Constant} = -\log t_r$$

or

$$\frac{t}{t_r} = \epsilon^{-(ea/GS)T}$$

or

$$t = t_r\epsilon^{-(ea/GS)T}$$

or

$$t = t_r\epsilon^{-(T/T_0)}. \qquad (109)$$

where T_0 has the same value as given by equation 108. To assist in numerical applications of this equation Fig. 48 gives the value of $\epsilon^{-(T/T_0)}$ for various values of T/T_0.

Example 48. Draw the cooling curves for both the oil and the windings, for the same transformer as that covered by the preceding examples.

From equation 109, for the oil,

$$t = 43^{-(T/208)}$$

which is the relation between time T and the temperature rise t of the oil, as the transformer cools, starting with the oil at 43 degrees C. The cooling curve is shown in Fig. 47.

OVERLOAD TEMPERATURE RISE CURVES

Having plotted, with the use of equation 107 or from tests, the curves showing the increase of the temperature rise of the oil and windings with time, starting with 100 percent load, the corresponding curves for any other load may readily be determined. The calculation of overload temperature curves from the 100 percent load values is based on the fact that the temperature rise of the oil is proportional to the total loss in the transformer and temperature rise of the windings above the oil is proportional to the copper loss. Suppose it is desired to draw the temperature rise curves for 125 percent load. Assume that after eight hours operation at 100 percent load the oil rise is 34 degrees C. and the rise of the windings is 45 degrees. If the normal iron loss of the transformer be 33 percent and the normal copper loss be 67 percent of the total loss, a load of 125 percent will produce a copper loss of 104 percent, the copper loss increasing as the square of the load. The total loss for this load will then be 137 percent, therefore the temperature rise of the oil will be $1.37 \times 34 = 46$ degrees C. The rise of the copper above the oil will be $(1.25)^2 \times 11 = 17$ degrees C. The temperature rise of the copper will then be $46 + 17 = 63$ degrees C. This procedure can be repeated for a sufficient number of points to enable the construction of the complete temperature curve. The temperature rise curve of the windings for no-load, that is, the temperature rise in oil due to the core loss only, can be determined by the same method. If the normal core loss is 33 percent of the total loss, the temperature rise of the oil and the winding also will, in this case, be $0.33 \times 34 = 11$ degrees C.

The curves given in Fig. 49, which show the rise of the oil

at no-load and at 100 percent load and the copper at 100, 125 and 150 percent loads, have been determined by test, and they check up very closely with the theoretical curves as outlined above. The temperature rise of the oil is determined by a thermometer and of the windings by the

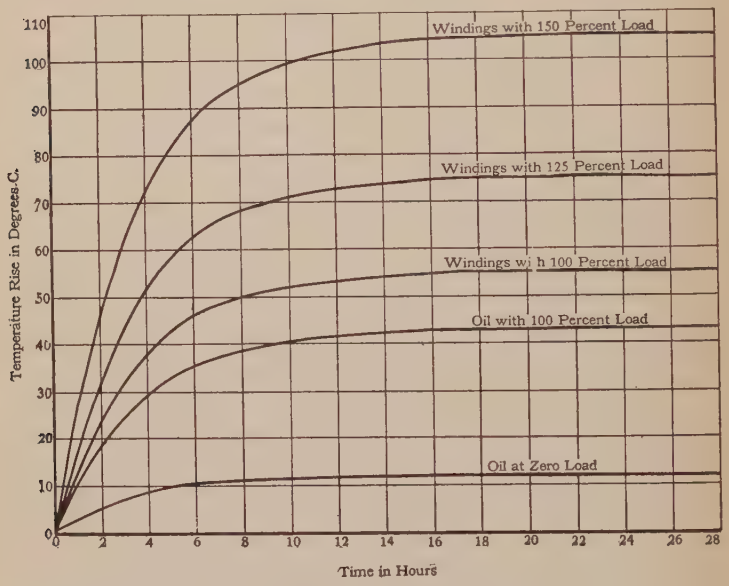


FIG. 49. Typical temperature curves with different loads on the transformers.

increase of resistance method. These curves do not represent any particular size of transformer, but rather that characteristic of a line of distributing transformers of from one to 50 k.v.a.

Example 49: As an example of the first case, suppose it is required to find the temperature rise of a transformer that is operating with no-load and then receiving a 100 percent load for four hours, followed by a 150 percent load for two hours. From the temperature curves in Fig. 49, it is seen that the windings have approximately a 12 degree rise, due to the iron loss only. To obtain the temperature rise of the windings after four hours

at 100 percent load, follow the 100 percent load temperature rise curve forward for a period of four hours from the point where it reached a temperature rise of 12 degrees C. This gives a temperature rise of approximately 38 degrees. When the 150 percent load is placed on the transformer it has a temperature rise of 38 degrees C., and will then continue to rise in temperature as indicated by the 150 percent load curve. Starting from the 38 degree C. rise, on the 150 percent load temperature rise curve, a transformer at the end of a two-hour period will have reached an approximate temperature of 63 degrees C. In this case its original temperature rise, due to its core loss, had little effect on its final temperature. Although it had a temperature rise of 12 degrees C. when the 100 percent load was started, this increased its temperature rise only two degrees C. at the end of the four-hour run at 100 percent.

SAFE MAXIMUM OPERATING TEMPERATURES

In determining the permissible load at which commercial transformers can be operated for a given time, and the permissible time for a given load, a maximum safe operating temperature should not be exceeded. It is generally understood that a safe maximum temperature for continuous operation is about 105 degrees C. Making an allowance of 10 degrees C. for hot spots and 40 degrees for the air temperature, gives a maximum temperature rise of 55 degrees C. This temperature rise may be exceeded for short periods of operation.

CHAPTER XI

PROPERTIES OF INSULATING MATERIALS

The study of the insulation of a transformer as a whole should be preceded by a separate examination of the properties of the insulating materials used, just as an acquaintance with the properties of copper and magnetic steel must accompany a thorough knowledge of the transformer.

Some of the properties of an ideal dielectric would be infinite resistance and perfect elasticity, in the sense that the energy required to establish an electric field through it would be returned as the field falls to zero. In other words there would be no energy loss in the dielectric when placed in an alternating electric field and the current required to maintain the electric field would be 90 degrees from the voltage in time phase relation. Such a current would be the wattless charging current of a condenser. In an actual dielectric there is an energy loss, made up of an I^2R loss and a loss due to the imperfect elasticity of the material. The latter is in some ways analogous to the hysteresis loss in a magnetic circuit, but the analogy fails if carried too far. If a curve be plotted for such a dielectric between current and electric field intensity, a further similarity to a magnetic circuit is that the curve will have some of the characteristics of a saturation curve. The curve may be considered as being composed of three parts: first, where the current is an approximately constant function of the field intensity and second where the current is growing at an increasing rate as the field intensity increases. This part of the curve represents the condition when the material approaches its breakdown point. As the dielectric fails the current increases

so rapidly that, third, the curve becomes practically parallel to the axis of current, or the saturation point is reached.

Another phase of the analogy between the dielectric and magnetic circuit is that the decreasing alternating-current resistance with increasing electric field intensity is similar to the changing reluctance of the magnetic circuit with increasing values of the magnetizing force. The impedance of the dielectric, however, is a very much more unstable quantity than the reluctance of the magnetic circuit.

Some of the most important properties of insulating materials are as follows, though the order in which they are given is not necssarily that of their relative importance:

1. Insulation resistance
2. Dielectric loss
3. Dielectric strength
4. Specific inductive capacity
5. Heat conductivity
6. Mechanical strength

Some of these characteristics are closely related with each other, for example, the presence of moisture in the material changes its insulation resistance, its dielectric loss and to a lesser degree its dielectric strength, without greatly changing its other properties.

INSULATION RESISTANCE

The minute spaces and capillary tubes in a solid dielectric contain varying amounts of moisture and gases. A conducting path through the dielectric is made up of a complicated arrangement of resistances and capacities in series and in multiple. The resistance of a given path through the dielectric is called its insulation resistance, and the current in phase with the voltage producing the electric field is called the resistance current. Direct current is used to measure insulation resistance as the charging current with

alternating current is usually relatively large and overshadows the resistance current. Energy measurements are necessary as well as voltage and current to determine the effective alternating-current resistance. Even with direct current the resistance varies with the applied voltage decreasing somewhat with increasing potential.

The principal conditions which influence the insulation resistance of solid dielectrics are

1. Temperature
2. Moisture

Fig. 50 gives a characteristic curve for fibrous materials, such as paper and cloth, showing the variation of the re-

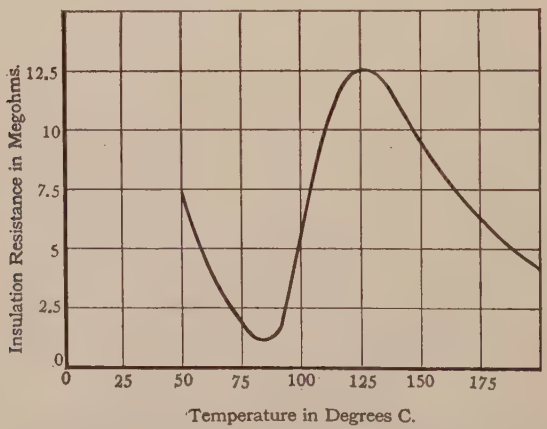


FIG. 50. Variation of insulation resistance of paper fiber with temperature.

sistance with temperature. The increased resistance at lower temperatures seems to be an inherent property of all insulating materials but is exaggerated by the presence of free moisture. It is probable that at the lower temperatures the moisture is in small isolated globules, which at higher temperatures vaporize and penetrate through the insulation,

thus lowering its insulation resistance. The greater the amount of moisture present the greater would be the difference expected between the hot and cold resistance measurements. If a piece of insulation is gradually heated and the moisture is free to escape, the relation between the temperature and the resistance will be like that shown in Fig. 50. The resistance at first decreasing because of the moisture being vaporized and again increasing because of the vapor being expelled at the higher temperatures. If a piece of insulation is carried through a series of such cycles, the amount of moisture remaining at the end of each cycle would become less and less. The change in the resistance between the extremes of temperature would therefore become smaller, being then largely that due to the intrinsic quality of the resistance.

After the expulsion of the moisture, the material begins to change in nature and ultimately begins to carbonize and behaves as a high-resistance conductor; the resistance then falling rapidly to a very low value. While this curve is typical of all insulating materials, the exact temperatures at which the minimum and maximum values occur depend mainly on the area of the electrodes, the amount of moisture content and the facility with which it may escape, and on the chemical and physical changes that occur due to the temperature.

DIELECTRIC LOSS

In general, the dielectric loss for a given material increases with

1. Voltage
2. Moisture
3. Temperature
4. Frequency

The variation of the dielectric loss for dry, oil-saturated, insulating board is shown in Fig. 51[1] expressed in watts per

[1] See article on "Dielectric Losses in Insulating Materials" by Mr. C. E. Skinner, in *The Electric Journal* for July, 1917, p. 260.

cubic centimeter, with the field intensity expressed in kilovolts per centimeter. The power-factor curve is also given, and the low values mean that there is very little I^2R loss in the material, which in turn indicates that there is not much

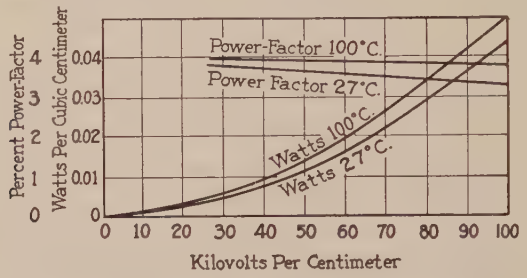


FIG. 51. Variation of dielectric loss of dry, oil-saturated fullerboard with field intensity at 60 cycles. The sheet was tested in oil immediately after being dried and saturated with oil by the vacuum-pressure method.

moisture present. The absorption of a slight amount of moisture would greatly increase the I^2R loss and consequently increase the power-factor. When the dielectric loss is largely I^2R loss due to the presence of moisture, the power-factor increases with increase of the electric field intensity, because the increase in the I^2R part more than overbalances the increase in the so-called hysteretic element. On the other hand, with dry material the increase of the hysteretic element of the loss may overshadow the I^2R loss and the power-factor would then decrease with increase of the field intensity.

The reason for the increased loss at higher temperature is largely due to the reduced resistance caused by the increase in temperature. This increase is therefore more marked with materials which contain a considerable amount of moisture.

The dielectric loss varies with the frequency, the hysteretic element being approximately proportional to the frequency and the I^2R element, of course, being constant. If

the material contains moisture and is operated at a fairly high temperature, the I^2R element of the loss will overshadow the hysteretic element and the loss for 25 and 60 cycles will not be far apart. If on the other hand the hysteretic element is comparatively large, a greater percentage difference will exist between the 25- and 60-cycle dielectric loss measurements.

For dry material where the I^2R element of the loss is smaller, the dielectric loss in watts per cubic centimeter for a given temperature may be approximately expressed by the relation

$$\frac{\text{Watts}}{\text{Cubic Centimeter}} = KfE^2,$$

where K is a constant which varies with the insulating material and the temperature at which it operates, f is the frequency in cycles per second and E is the voltage in kilovolts. For the curve shown in Fig. 51 and for a temperature of 100 degrees C., K has a value of approximately 8.5×10^{-8}. If the material contains moisture, the rate of increase of the toal loss will vary greatly, and may for a limited range increase approximately as the square of the frequency.

When the dielectric loss ceases to increase approximately as some constant function of the field intensity, the approaching breakdown of the dielectric is indicated. This disclosure of the loss measurements is of more importance than the actual value of the loss in watts. The particular value of the dielectric loss measurement is to determine what conditions or treatment will permit the highest possible value of the field intensity in a given material, without approaching the critical value.

DIELECTRIC STRENGTH

The main factors affecting the dielectric strength of materials for a given electric field intensity are:

1. Temperature
2. Chemical change

Apparently the failure of a dielectric under electrostatic stress is because the material is stressed to such a degree that it is mechanically ruptured. An example of a case where there is little change of the material except that resulting from the stress is the breakdown of clean dry oil. Usually the breakdown in solid materials occurs at a temperature which has itself caused some deterioration and thus makes possible the mechanical rupture at a voltage which would not otherwise have caused injury. The temperature may result from the dielectric being heated from an outside source, or from the dielectric loss within the material. An example of this type of failure would be that of paper fiber under conditions where the heat from the dielectric loss could not be readily dissipated. The temperature of the dielectric would continue to rise, with a fixed impressed voltage, until the material had become so injured that failure might occur. In other words, the temperature conditions are cumulative, and a voltage which would not break down the material in a short time might do so if continued for a longer period. There also may be a condition analogous to the fatigue of materials when subjected to alternating mechanical stresses, which has an influence on the diminution of the dielectric strength with increased time of application of the testing voltage.

Thus all factors which affect the dielectric loss and the rate of dissipation of the heat would have an influence on the exact value of the breakdown voltage. With a low dielectric loss and good conditions for dissipating the heat, the question of the length of time the voltage is applied will have a relatively small effect. Therefore, the effect of time is more marked with thick masses of material and in such cases there is an apparent reduction in the dielectric strength with an

increase of the thickness of the material. The frequency of the alternating voltage affects the dielectric value somewhat, mainly through change in the dielectric loss. Surges caused by switching or lightning, whose voltage is many times higher than the breakdown voltage of the insulation, may be applied without rupture if the time period is very short. If the voltages are sufficiently high, rupture may occur at once, and if the voltage is higher than the 60-cycle breakdown voltage, the insulation will be damaged probably by mechanical tearing. The effect is cumulative and a sufficient number of such surges will cause a breakdown. The shorter the time of application, the greater the number of surges required to cause a breakdown at a given voltage. The insulation of transformers is often gradually destroyed in this way by lightning, each stroke contributing toward breakdown, probably by extending local ruptures.

If the material contains oxidizable substances or chemically unstable matter, the breakdown may be the result of chemical change. In most commercial insulating materials, it is probable that all of these conditions affect the disruptive voltage.

There are a number of other minor factors which have an influence on the dielectric values, for example, the nature of the surrounding medium, size and shape of the electrodes, and time-rate of application of the disruptive voltage. Tests are useless for comparing insulating strength values unless made upon some definite basis.

SPECIFIC INDUCTIVE CAPACITY

Since a good dielectric should have a low charging current, it follows that it would also have a low specific inductive capacity. With two dielectrics in series, across which voltage is impressed, the voltage across each will be inversely as the dielectric constants of the two materials. When several dielectrics are used in series, the one having

the lowest dielectric constant takes more than its proper share of the total voltage, and if it also has a low dielectric strength it may break down, even if it would have been strong enough without the presence of the other material. Thus placing a plate of glass between two electrodes will cause a spark to pass through the remaining air with a lower voltage than would be required to produce a spark in the absence of the glass. This fact is of special im-

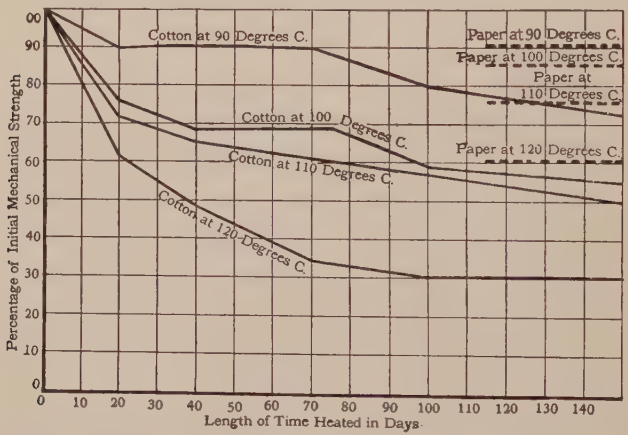


FIG. 52. Effect of temperature on the mechanical strength of paper and cotton fiber heated under oil.

portance in high-voltage work, for air has both a lower dielectric constant and a lower dielectric strength than solid insulating material. Air bubbles reduce the effective dielectric strength, and may with alternating voltage contain corona which will soon damage the solid dielectric. For this reason, air should be excluded from the insulation of coils designed for high voltages, either by impregnating them with a suitable solid compound or by using them immersed in insulating oil.

The dielectric constants of some of the common insulating materials are

Material	Dielectric Constant
Oil	2 to 2.5
Dry paper	2.6
Mica	5 to 7

MECHANICAL STRENGTH[2]

The curves, Fig. 52, show that the mechanical strength of paper and cotton fiber tend to reach a constant value after

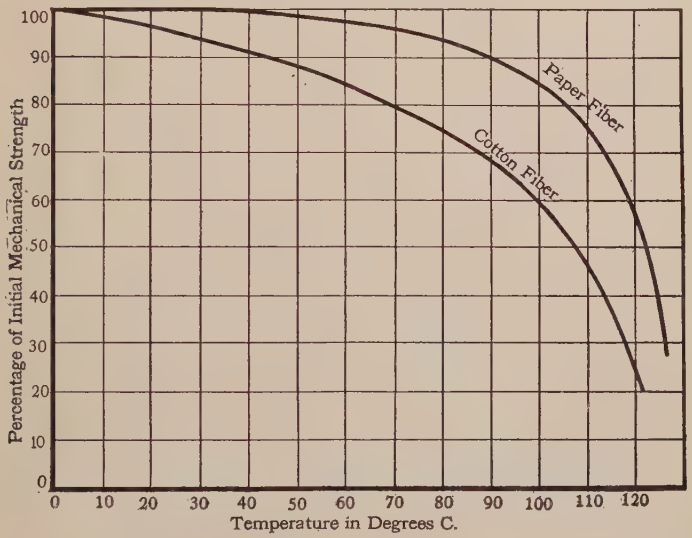


FIG. 53. Effect of different temperatures on the mechanical strength of paper and cotton fiber. The temperature conditions were continued until the mechanical strength showed no further change.

prolonged heating in oil at a definite temperature. Fig. 53 shows the mechanical strength of the insulation as a variable dependent on the temperature.

[2] See *The London Electrician* for July 20, 1917.

CHAPTER XII

TRANSFORMER INSULATION

THE AMOUNT and arrangement of the insulation in a transformer has almost as much of an empirical as a theoretical basis. The amount of copper in the electrical circuits and iron in the magnetic circuits is a matter of calculation when the characteristics of the materials are known and certain results are to be secured. On the other hand, the amount of insulation to be used in a given place quite often depends upon its mechanical characteristics, and the stresses which it must withstand during winding.

INSULATION TESTS ON COMMERCIAL TRANSFORMERS

Usually, the tests made on commercial transformers are in accordance with the standardization rules of the American Institute of Electrical Engineers. Disruptive tests are made between the high- and low-voltage windings and between the windings and ground. Similar tests are also made between the several parts of either the high- or low-voltage windings, when composed of two or more electrically separate parts.

While an over-potential test is made (at a frequency higher than normal to prevent over-excitation of the magnetic circuit), the voltage between turns of the winding is not sufficient to really test the insulation to its normal breakdown value. The voltage developed at the terminals of the coils limits the extent to which the over-potential test can be carried. It has been proposed that some test be developed by which a surge could be thrown on the transformer, by the use of a discharge from a condenser, duplicating in some measure the transient conditions in service, but so far such a test has not been standardized.

DIELECTRIC CHARACTERISTICS OF THE INSULATED TRANSFORMER

The practical value of insulation resistance measurements on transformers is that they give some idea of the amount of moisture or dirt present, a low insulation resistance value being serious largely because it indicates a probable low dielectric strength of the insulating material and a possibility of failure due to high dielectric loss. Further, an insulation resistance measurement does no harm to the transformer, while a failure under the disruptive test may injure the insulation at the point of failure.

Insulation resistance varies inversely as the cross-section of the insulating material, and this is the reason why high-voltage transformers may have a lower resistance than lower voltage transformers of the same rating. The insulation contact surfaces between windings and iron and between coil windings of high-voltage transformers may overbalance the lesser thickness of insulation used in low-voltage transformers. Dry transformers have resistances of from 200 to 2,000 megohms..

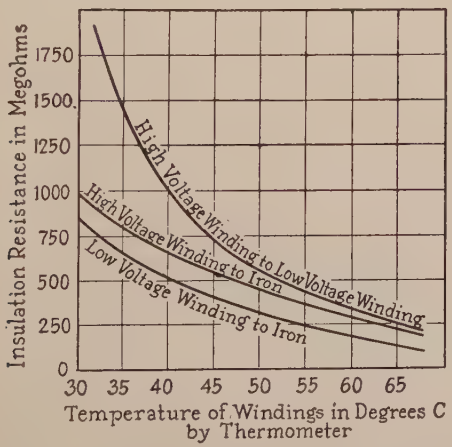


FIG. 54. Variation of the insulation resistance with temperature of a 2,300 to 230 volt, 60 cycle distributing transformer.

The insulation resistance of a small 2,300–230 volt, 60-cycle distributing transformer at various temperatures is shown in Fig. 54. These curves were made from a transformer which passed all standard insulation tests and was therefore suitable for service. The tests were made with the case filled with oil and the transformer heated to approximately 70 degrees C. It was then allowed to cool, taking resistance measurements as the temperature dropped.

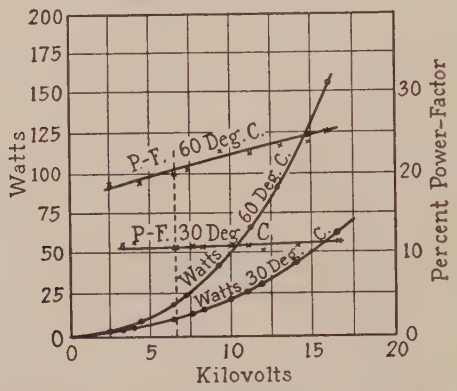


FIG. 55. Dielectric losses in a 150 k.v.a., 60 cycle transformer. The vertical line indicates the normal high-tension operating voltage of the transformer.

The dielectric loss for a 150 k.v.a., 6,600–440 volt, 60-cycle transformer is shown in Fig. 55. The test was made under the same conditions used for the dielectric strength test, that is from the high-voltage to the low-voltage winding, with the latter connected to the magnetic circuit. The dielectric loss with this connection is greater than the loss would be with the transformer in actual service, because all of the windings are at the same potential. In service the potentials of the windings with regard to each other change from point to point in the coils. The losses in actual operation will be about one-third of the losses indicated on the

curve for the same voltage, but changing somewhat with the particular connection used. However, the dielectric loss in a transformer under the most unfavorable conditions is very small as compared with the iron and copper losses.

The dielectric strength of the insulation between the various parts of a transformer must be such that the conditions arising in service will be met with a reasonable factor of safety. The greater part of the insulating materials used in transformers is cotton and paper fiber.[1] It is therefore reasonable to assume that a transformer which has operated for a period of from 100 to 200 days at full load runs little risk of later burning out under the same conditions.

Since the greatest mechanical stresses are placed on the insulations when the coils are being wound, after the winding and assembling are accomplished, a considerable diminution of the mechanical strength of the insulation is then permissible. Perhaps one-half or one-quarter of the original strength would suffice, particularly since the insulating materials are re-enforced by the impregnation treatment.[2] The heat-resisting properties imparted by the impregnation treatment form another factor of safety in addition to that possessed by the unimpregnated paper and cloth.

IMPREGNATION TREATMENT

The main purpose of the impregnation treatment of transformers is to remove the moisture and seal it out with a compound which is itself an insulator. Before the impreg-

[1] Whose mechanical characteristics under operating conditions are shown in Figs. 52 and 53. Fig. 52 shows that, for various temperatures, constant conditions are reached in the neighborhood of 100 hours.

[2] If say forty percent was sufficient, from Fig. 53, it is seen that this percentage of the initial value of the mechanical strength is retained with cotton heated in oil at 114 degrees C., and paper heated in oil at 125 degrees C. These temperatures are higher than those usually given, even when account is taken of the fact that the actual maximum temperature is approximately ten degrees higher than that measured by the increase of resistance method.

nation treatment was used, the same results were partially secured by drying and treating the insulating material used. This method does not give as good results as impregnation, because the treatment is confined practically to the surface of the material. With the impregnation treatment, the subsequent accumulation of moisture is largely confined to the outer surfaces, which is to be expected when the pores of the paper and cloth as well as the open spaces between are filled with the compound.

The impregnating compound can be liquefied by heat, forced into the coils, and on cooling it forms into a solid mass. The wires are thereby held in position, thus preventing motion and abrasion in service, and the heat conductivity of the coil is materially increased. Tests also show that the impregnated coils show no deterioration at temperatures where plain cotton covering on conductors is injured.

The materials which can be used for impregnation are varnishes, shellacs, asphalts and resins, since they can be applied in the form of liquids which later solidify. Varnishes and shellacs contain a considerable percentage of volatile solvents which must be driven off before the material reaches the solid state. These materials are therefore seldom used to impregnate coils because of the difficulty in expelling these solvents. If the solvents could be satisfactorily expelled, the coils would be full of pores where the solvent had worked through. This leaves the asphalts and resins as the materials best suited for impregnation. Since all asphalts are soluble in oil to a certain extent, they are not satisfactory for use in oil-insulated transformers. The oil-proof compounds are made from the resinous gums, compounded so as to give the desired hardness, melting point and penetrating power. The gums used come generally from Asiatic ports, and are excretions from trees belonging to the same family as the pine, from which the common rosin is secured.

In choosing insulating materials for impregnating treatment the readiness with which they absorb the compound should be considered. Cotton in the form of cloth, tape, or wire covering is easy to apply, and has good mechanical strength but does not have a high dielectric strength. For these reasons cotton fiber is particularly good as an insulating material where the impregnating treatment is to be used, serving as a supporting material for the impregnating compound, which has poor mechanical characteristics, but high dielectric strength. Varnished materials, fiber and similar substances are practically impervious, even to the most fluid of the gums.

Large spaces between wires or sections of a winding are difficult to fill. It is important in the design of the coils that a path be always left for the gums to enter such spaces, as, if they are deep within the coil, there will otherwise be no path for the gum except through capillary spaces too small to admit a large amount. If they are near the outside, before the coil can be cooled the gum will have run out.

In the process of impregnating, the coils are first dried, either in a separate oven or in the impregnating tank. They are then placed in the tank and heated to a temperature high enough to make the gum fluid, but not high enough to injure either the gum or the insulation of the coil. A vacuum pump is then applied; usually from one to six hours for an ordinary coil provided it is thoroughly pre-heated. After the vacuum has drawn the moisture from the coils, the impregnating compound, previously heated to a liquid condition, is drawn into the tank by the vacuum. Pressure is then applied and maintained until the desired degree of impregnation is secured. With a very fluid gum, high temperature and high pressure, two hours under pressure may be sufficient, but under adverse conditions ten hours may be required. The medium furnishing the pressure must be dry before entering the tank, and the pressure generally used is from 80 to 100 pounds per square inch.

PROPERTIES OF INSULATING OIL

Transformer oil should have the following characteristics:[3]

1. High dielectric strength
2. Low viscosity
3. Freedom from deposit
4. High flash point
5. Low evaporation

The dielectric strength of clean dry oil at 25 degrees C. is usually not less than 22,000 volts when tested between one inch brass disks set 0.10 inch apart, and changes somewhat with temperature. The dielectric strength is a maximum at a temperature between 60 and 70 degrees C. and decreases for temperatures both below and above this point. It is an important fact that the oil is at its best dielectric value at or near its normal working temperature. The resistance of oil decreases with temperature increase as does that of other insulating materials. The presence of as small an amount of water as 0.01 percent is sufficient to seriously impair the dielectric strength of the oil. The exact composition of a mixture of oil and water is a little obscure, but it seems that, with clean oil, the mixture is more or less temporary, and that the particles of water will finally settle out. In an electrostatic field the small drops are drawn together and tend to form larger ones by collision, and in transformers they will generally be attracted to the points under the greatest stress. If the oil contains fibers of foreign matter, as practically all oils do, the fibers will absorb the particles of water and thus tend to retain them in the oil. Dirt in oil may have an effect very similar to moisture and the electric field may cause the small conducting particles to bridge between the electrodes.

[3] See Report of Committee on Electrical Apparatus of The National Electric Light Association.

The presence of acid or alkali in insulating oil is not permissible, because they reduce its dielectric strength, as well as act destructively on the materials of the apparatus. An oil practically neutral is preferable, and it should contain no free sulphur which in its free form attacks copper. All petroleum oils contain sulphur compounds which, however, are not objectionable.

Since oil is usually used in transformers as a cooling as well as an insulating medium, its viscosity is important. The more sluggish the oil, the slower will be its circulation and the transfer of heat will be correspondingly slow. Heavy oil will not circulate freely through the oil ducts of the windings, with the result that there is a high temperature gradient between the oil and windings. The viscosity of oil used for transformers is ordinarily about 50.0 seconds Saybolt at 40 degrees C.

Deposit from oil is objectionable because it clings to the windings and fills the ducts. The cooling of the apparatus is thus affected by any deposit which occurs under normal conditions of operation. Such a deposit also renders the oil more sluggish and thus further affects its cooling action. A deposit is also an indication that chemical decomposition is taking place in the oil. Its formation is a function of the temperature and length of time the oil has been used, but at ordinary temperatures there is a tendency to form a deposit in the presence of an electric field of high intensity. Oils differ as to the temperature at which a deposit will appear. A satisfactory oil should not show a deposit or any change, except possibly a darkening of color, when operated continuously under the temperature conditions experienced during actual operation. Heating the oil gradually up to a temperature of from 200 to 230 degrees C. in one hour should not produce any other effect than a darkening of color.

Since oil in transformers may occasionally be subjected

to high temperatures caused by overloads, or locally by a lightning discharge, or by corona effects, it is important that its characteristics be such as to prevent an explosion or fire even under those abnormal conditions. The usual criteria of the inflammability of oils are their flash and fire tests, usually taken with the open cup. The values for the open cup, however, differ only slightly from those taken with the closed cup, being a few degrees higher. A grade of oil found satisfactory for transformer work has a flashpoint around 140 degrees and a firepoint in the neighborhood of 155 degrees C. It is obvious that an explosive mixture is not formed with this oil until the temperature attained by an overload on the transformer is greatly in excess of normal operating temperature. When this oil is slowly heated experimentally in a closed vessel, the proportions of oil and gas spaces approximating those existing in a transformer, and an arc is formed at temperature intervals in the gaseous mixture, the lowest temperature at which an actual explosive mixture can be obtained is about 150 degrees C. However, at room temperatures it is possible to secure an explosive mixture by maintaining an arc, either below or at the oil surface. There is no danger therefore of an explosion from a lightning discharge or corona effects, unless there is an actual arc produced at the surface or beneath the oil which can evaporate enough of the oil to produce an explosive mixture, and give at least a localized temperature in excess of that of the flashpoint of the oil. Furthermore, there would be no great advantage resulting from increasing the flashpoint of the oil say 25 percent, since if the explosion occurs only when there is an arc present, it would take but a slightly longer time for the heavier oil to reach the explosive condition than it does with the lighter oil.

Light colored oils have some advantage since they permit the superficial examination of transformer connections by

the aid of an incandescent lamp, without removing the apparatus from the oil.

All insulating oils evaporate slowly even at operating temperatures, and this is more pronounced in oils of relatively low viscosity. A satisfactory oil should not show undue evaporation under normal operating conditions.

CHAPTER XIII

REACTORS

A REACTANCE coil or reactor is composed of one or more turns of a conductor linking with a magnetic circuit, which may be air or metal. When a voltage is impressed on such a coil, a current flows which may be thought of as the magnetizing current of its magnetic circuit. An ideal reactance coil would be one in which the current flowing with a given impressed voltage would be 90 degrees behind the voltage in time-phase relation. Since there is a voltage drop in the conductor which is in phase with the current, such a condition cannot exist, and a reactance coil is in reality an impedance coil with the reactance element predominating.

The rating of a reactance or impedance coil at a given frequency might be given in ohms, but as such a rating gives no clue to its current-carrying capacity it is usually given in k.v.a. The rating is the product of the voltage, expressed in kilovolts, required to force a given current through the coil and the value of the current in amperes.

The reactance of a coil with an air magnetic circuit is small, so that a large current is established by a small impressed voltage. The reactance is small because the reluctance of the magnetic circuit is large and a large current is required to establish a given flux density and thus produce a given counter electromotive force. Therefore, in order to produce any considerable counter electromotive force, a relatively large number of turns or a large area of the magnetic circuit is required. On the other hand, with an iron magnetic circuit a large voltage is required to send a small current through the coil. By putting a gap in the iron circuit,

its reluctance is increased so that the ampere-turns required to establish a given magnetic density are also increased. Any desired step between a circuit of air and one of iron may be secured by placing air-gaps in the circuit. There are therefore three types of reactance coils as follows:

1. With a magnetic circuit of air.
2. With a magnetic circuit of iron.
3. With a magnetic circuit of iron having air-gaps.

One of the main problems in designing air reactors is to keep the eddy-current loss in the conductor to a reasonably low value. The problem in this respect is similar to that described in chapter II under Copper Loss.

WITH A MAGNETIC CIRCUIT OF AIR

The voltage drop across a reactance coil without iron is proportional to the current flowing, because air, which has a constant permeability, forms the magnetic circuit. For applications where the voltage is required to vary directly with the current flowing, up to such a value as will heat the copper to the permissible limit in a relatively short time, the air reactor is preferable. Air reactors have a number of applications, among them being:

1. Standards for precise measurements.
2. Coils for reactance circuit-breakers.
3. To counterbalance the effect of distributed capacity in transmission lines.
4. In transmission lines, to protect against lightning disturbances.
5. To protect generator and feeder circuits against short-circuits.

As far as the calculation is concerned the simplest form of a reactor is that of a long solenoid shown in Fig. 56. For

such a coil the inductance, in abhenrys, is:

$$L = \frac{4\pi^2 a^2 T^2}{l},$$

where a is the mean radius of the coil, l its length in centimeters and T its number of turns. When L is expressed

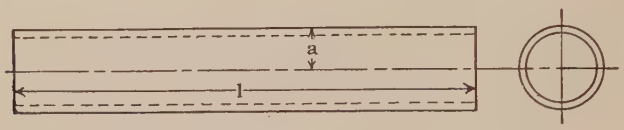


FIG. 56. Reactor in the form of a long solenoid.

in henrys and the dimensions in inches, this equation becomes:

$$L = \frac{a^2 T^2}{10^7 l}. \qquad (110)$$

While there is a considerable error in this formula due to the end effect, the variations in L due to changes in l are almost exactly proportional to the change in l. Therefore, it may be used without error for calculating variations of L due to changes in l. The counter e.m.f. of this coil is $2\pi fLI$, where f is the frequency and I the current flowing in amperes. Therefore, by the use of equation (110):

$$E = \frac{2\pi}{10^7} \times \frac{fa^2 T^2 I}{l} = \frac{6.28}{10^7} \times \frac{fa^2 T^2 I}{l}. \qquad (111)$$

Example 50: What is the voltage and k.v.a. rating of a reactor which has the following constants? $f = 60$; $a = 8.82$; $T = 400$; $I = 53$; $l = 50$.

From equation (111),

$$E = \frac{6.28}{10^7} \times \frac{60 \times 8.82^2 \times 400^2 \times 53}{50} = 496 \text{ volts.}$$

With a current of 53 amperes the rating is $(496 \times 53)/1{,}000$ or 26.2 k.v.a.

The simple form of solenoid reactor shown in Fig. 56 does not give a maximum reactance value for a given amount of material and therefore practical reactors, particularly of the feeder circuit protective type which are usually of fairly large size, are not built in this form and their calculation is therefore more involved.

The form of circular reactor shown in Fig. 57, where a section through the coil is a square, approaches more closely

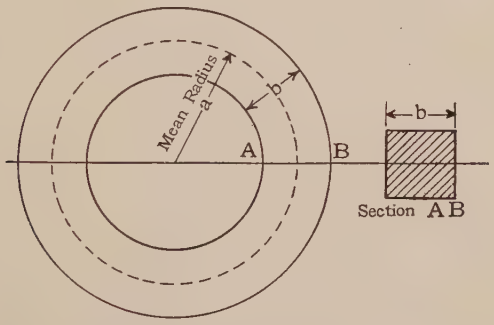


FIG. 57. Reactor with a magnetic circuit of air. Feeder circuit protective type.

to the most economical proportions. It has further been found that for a coil of this form, in order to obtain a maximum inductance for a given weight of conductor, the ratio $a \div b$ should be equal to 1.47. Substituting this relation in Weinstein's formulae given in the Bulletin,[1] which is applicable to a coil of square cross-section, gives:

$$L = \frac{aT^2}{238 \times 10^5} \qquad (112)$$

where L is expressed in henrys and a in inches.

[1] *The Bulletin of the Bureau of Standards,* Vol. 8, No. 1, gives a very extensive collection of formulae for the calculation of reactance coils of all proportions.

Since the counter e.m.f. of the coil is $2\pi fLI$, from equation (112):

$$E = \frac{faIT^2}{38 \times 10^5}. \qquad (113)$$

Example 51: What is the counter e.m.f. of the coil given in the preceding example when operating on a 60-cycle circuit and with a current of 53 amperes flowing?

From equation (113),

$$E = \frac{60 \times 8.82 \times 53 \times 400^2}{38 \times 10^5} = 1180.$$

If 1,180 volts will force approximately 53 amperes through this coil its capacity rating will be $(1{,}180 \times 53)/1{,}000$ or 63 k.v.a.

If practical considerations, as for example heating or floor space requirements for current limiting reactors, require a departure from the coil of square cross-section, a different formula must be used.

WITH A MAGNETIC CIRCUIT OF IRON

Reactors with a magnetic circuit of iron have a very limited field of application. Shunt coils for the individual lamps of a series incandescent lighting circuit form one special application of this class. For this case instead of starting from a formula for the inductance, it is preferable to start from the fundamental equation of the transformer, since the reluctance of the iron as compared to the air magnetic circuit is much easier to calculate. At a given voltage E and frequency f the coil must have its number of turns T, the square inches A_i of the cross-section of the magnetic circuit, and the magnetic flux density in gausses B so related as to satisfy this fundamental equation. For this case, equation (29) becomes:

$$E = \frac{f}{3.5 \times 10^6} TA_iB. \qquad (11$$

Example 52: What is the counter electromotive-force of a reactor with

an iron magnetic circuit, which has the following constants? $f = 60$; $T = 250$; $Ai = 1.5$; $B = 17{,}100$.

From equation (114),

$$E = \frac{60}{3.5 \times 10^6} \times 250 \times 1.5 \times 17100 = 110 \text{ volts.}$$

The current which this voltage will force through the coil when impressed on its terminals depends on the reluctance of its magnetic circuit. The value of this current may be calculated from the permeability of the material, but the most practical method is to calculate it from a curve similar to that in Fig. 14.

Example 53: What current will 110 volts force through the reactance coil given in the preceding example which has 4.9 pounds of iron in its magnetic circuit, if the magnetic induction is 17,100 gausses?

From a curve similar to that in Fig. 14, but which gives a greater range of values, the apparent watts is taken as 90 watts per pound. The total apparent loss is 90×4.9 or 440 watts and the current supplying this loss will be 440/110 or 4 amperes. If approximately 110 volts is required to force 4 amperes through a reactance coil, its rating will be $(110 \times 4)/1{,}000$ or 0.44 k.v.a.

Any change which affects the reluctance of the circuit will change the current which will pass through the coil. For example, slight gaps between the punchings where they butt together will change this reluctance. If the voltage impressed on the coil changes, the permeability of the iron changes. Therefore, the current flowing through the coil will not be proportional to the voltage impressed on its terminals. However, the current increases approximately as the voltage up to the point where the iron begins to be saturated, after which point the current will increase faster than the voltage.

It is practical to build reactors with a closed iron magnetic circuit only for small current ratings, due to the low reluctance of such a circuit, and consequently the practical application of these coils is very limited. When any considerable current capacity is required it is necessary to in-

crease the reluctance of the magnetic circuit by introducing air-gaps.

WITH A MAGNETIC CIRCUIT OF IRON HAVING AIR-GAPS

In this case the relation between the impressed voltage, frequency, number of turns, area of magnetic circuit and induction is the same as that given in equation (114). Also the current which will flow through the coil with a given impressed voltage depends on the reluctance of the magnetic circuit.

For reactance coils with air-gaps in the magnetic circuit, practically the entire magnetomotive force is concentrated on the gaps, since the reluctance of the iron parts of the circuit is relatively so small that it may be neglected. For the gap,

$$\text{Flux} = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{\frac{4\pi}{10} IT}{\frac{l_g}{A_g}},$$

where l_g and A_g are the length and sectional area of the gap respectively.

This becomes

$$H = \frac{4\pi}{10} \frac{\sqrt{2}}{2.54} \frac{IT}{l_g} = 0.7 \frac{IT}{l_g}, \qquad (115)$$

when H is expressed as the maximum value of the flux density in the air-gap and l_g is expressed in inches. In the

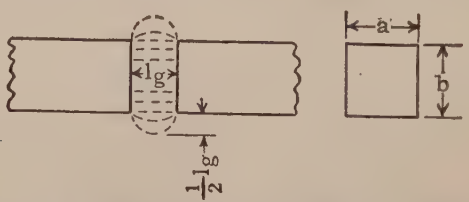


FIG. 58. Section of a magnetic circuit showing the fringing of the magnetic flux at an air-gap in an iron circuit.

gap the flux fringes out approximately as shown in Fig. 58, and therefore

$$H = \frac{ab}{(a + l_g)(b + l_g)} B,$$

where a and b are the dimensions of the iron section at the air-gap and B is the maximum value of the flux density in the iron. This expression may be put into the form

$$H = \frac{ab}{l_g(a + b) + ab} B$$

if l_g^2 which is a small quantity be neglected. Substituting this value of H in equation (115) gives

$$B = \frac{0.7IT}{l_g}\left[1 + \frac{(a + b)l_g}{ab}\right]. \qquad (116)$$

The effect of this fringing is not considerable for small gaps in the neighborhood of one eighth of an inch or less, but becomes an increasingly important factor as the length of the gap increases.

Example 54: What current will a voltage impressed on a reactance coil such as to develop a flux density of 10,000 gausses in the iron, force through, under the following conditions? $T = 400$; $a = 2.7$; $l_g = 0.1$; $b = 5.4$.

From equation (116),

$$I = \frac{10000 \times 0.1}{0.7 \times 400\left[1 + \left(\frac{2.7 + 5.4}{2.7 \times 5.4}\right) \times 0.1\right]} = 3.4 \text{ amperes.}$$

If approximately 1,000 volts are required to force 3.4 amperes through a reactance coil, its rating will be $(1{,}000 \times 3.4)/1{,}000$ or 3.4 k.v.a.

With a given number of ampere turns it is often important to determine the length of the air-gap which will give a reluctance sufficient to prevent the magnetic density in the iron from exceeding a specified value. For this purpose

equation (116) may be put into the form

$$l_g = \frac{1}{\frac{B}{0.7IT} - \frac{a+b}{ab}}. \qquad (117)$$

Example 55: What is the length of air-gap in the magnetic circuit required to give a flux density of 10,000 gausses in the iron, under the following conditions? $IT = 1{,}400$; $a = 2.7$; $b = 5.4$.

From equation (117),

$$l_g = \frac{1}{\frac{10000}{0.7 \times 1400} - \frac{2.7 \times 5.4}{2.7 + 5.4}} = 0.14 \text{ inch.}$$

In general if the voltage of a reactor is not required to vary directly with the current for values considerably in excess of its normal full-load current rating, the coil with an iron magnetic circuit will be cheaper. The iron magnetic circuit reactor also has the advantage that the magnetic flux is confined to the iron circuit, while the air coil sets up a strong field in its immediate neighborhood.

The design of the reactance coil for a minimum cost, with the iron and copper working densities fixed, is worked out exactly as for a transformer; taking into account that the volume of the air-gap is to be subtracted from the total volume of the magnetic circuit.

CHAPTER XIV

CONNECTIONS FOR VOLTAGE TRANSFORMATIONS

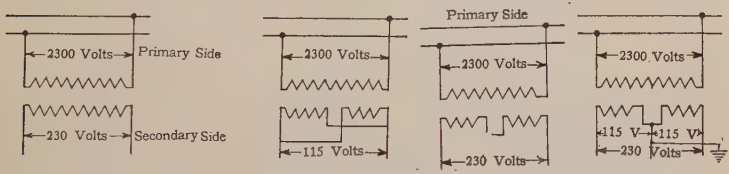
THE following are some of the most important transformer connections for voltage transformations:

1. Single-phase.
2. Three-phase by the various combinations of the star and delta.
3. Three-phase open delta.
4. Three-phase T.
5. Interconnected star.

This discussion does not go into detail regarding the vector relations of the voltages and currents, nor the k.v.a. of transformer parts required by the different arrangements, but merely shows the connections required to give the transformations.

SINGLE-PHASE CONNECTIONS

The simplest transformer arrangement is that shown in Fig. 59, since both the high- and low-voltage windings are



FIGS. 59 and 60. The common methods of connecting low-voltage, single-phase transformer secondaries.

made up of one part without taps. The winding may be divided into parts and these parts connected for the differ-

ent operating conditions. For example, most distributing transformers are made with two low-voltage coils, which may be connected as shown in Fig. 60, for parallel, series and three-wire operation. The series-parallel connection permits operation at full output at two voltages, one of which is double the other. The three-wire connection is extensively used, as it permits simultaneous feeding a combined lighting and power load. In this case the motors are connected to the two outside wires, and the lights between the outside wires and the neutral. The neutral of the three-wire connection gives a convenient point for grounding, when it is desired to connect the secondary of the transformer to the ground.

Taps are usually provided on transformer windings of the higher voltage classes, the main purpose of the taps being to obtain a given secondary voltage, when the transformer

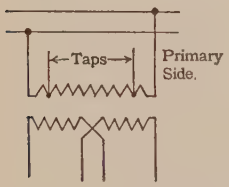


FIG. 61. Taps on the line ends of the winding.

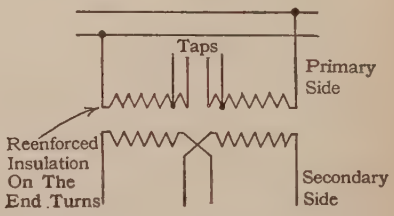


FIG. 62. Taps at the middle of the winding.

is used at points where the primary voltage is low, due to line drop or other reasons. The taps may be located on the transformer winding by either of the methods shown in Figs. 61 and 62. The one shown in Fig. 62 is used for the higher voltage transformers, where the insulation is increased on the end turns. With the connections shown in Fig. 62, this additional insulation need extend back but a short distance, while if the taps are on the line ends of the winding, the insulation must extend beyond the taps.

THREE-PHASE DELTA AND STAR CONNECTIONS

With the delta-delta connection, the windings are connected to the mains as shown in Fig. 63. The line voltage

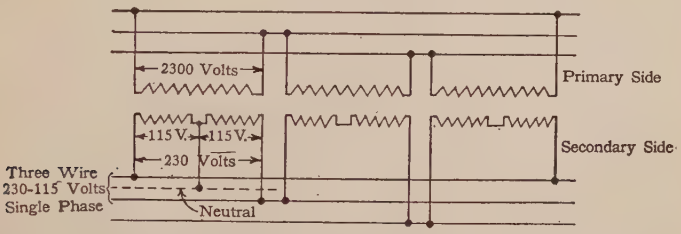


FIG. 63. Delta-delta connected transformers for three-phase transformation, showing a three-wire, single-phase secondary circuit on one transformer.

is equal to the individual transformer voltages, and is commonly denoted as the delta voltage to distinguish it from the voltage of the group connected in star. Similarly, the line current must be distinguished from the current flowing in the windings of the transformers. When speaking of the voltage and current of a three-phase system, the delta voltage and star current are understood.

Where 2,300 to 230 volt transformers are connected delta-delta for a three-phase transformation, one of the units is sometimes made larger than the other two, and the middle point of the low-voltage winding is connected as shown in Fig. 63, giving a 230–115 volt single-phase three-wire lighting circuit. If one transformer or one phase of a three-phase transformer is disabled, the other two may be used in open delta.

The delta-star connection shown in Fig. 64 is used to a considerable extent, since a fourth wire may be led from the neutral point of the low-voltage windings, thus giving a convenient and economical distributing system. Single-phase service is obtained by connecting between any line and the neutral, while for three-phase work the line wires

are used, the voltage being $\sqrt{3}$ times the single-phase voltage.

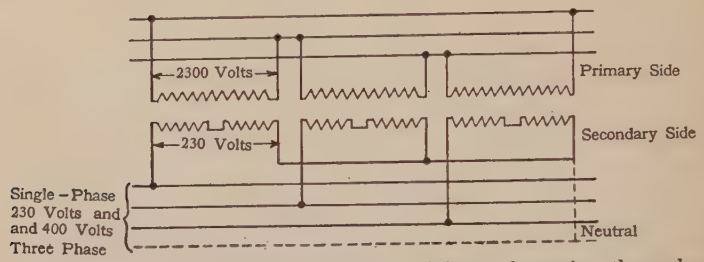


FIG. 64. Delta-star connected transformers, giving a four-wire, three-phase secondary circuit.

With a star-delta connection a neutral point is made available in case it is desired to ground the primary side of the transformer, or use the four-wire system. The four-wire system is mostly used for a combination of motor and lighting loads, the lighting service being operated from the 2,300 volt phase voltage and the power service from the 4,000 volt line voltage, as shown in Fig. 65. Transformers

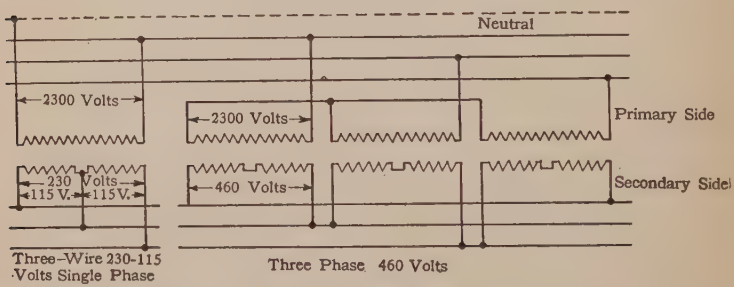


FIG. 65. Star-delta connected transformers from a four-wire primary circuit, showing also the connections of a single-phase, three-wire transformer from the same primary.

are sometimes designed so as to be suitable for either delta-delta or delta-star connection, in order to permit an increase in the capacity of a transmission line by raising the line

voltage, which can be accomplished by changing the connections from delta to star on the high-voltage side. Such transformers are necessarily more expensive than they would be if designed for straight delta-delta, and used at the lower voltage only, because they must be insulated to withstand the higher line voltage. With the star-delta connection of three single-phase transformers, one unit may be cut out, and emergency service be maintained as shown in Fig. 66. The capacity of the group is in this case re-

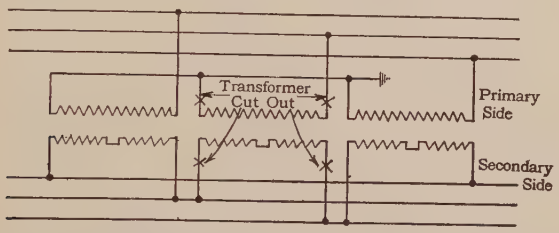


FIG. 66. Emergency connection for three-phase transformation when one transformer of a star-delta bank burns out and the neutral of the power source is grounded.

duced to 58 percent of its original value. The neutral connection on the high-voltage side should preferably be made

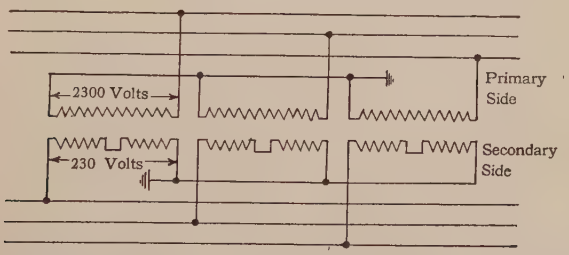


FIG. 67. Star-star connected transformers with the neutrals of both primary and secondary circuits grounded.

through a wire, but can be made by solidly grounding the neutral, provided the neutral of the source of supply is also grounded.

The star-star connection of three single-phase transformers, or of a three-phase unit as shown in Fig. 67, is not to be recommended unless their neutrals are grounded.

OPEN DELTA CONNECTION

When three single-phase units or a three-phase shell-type transformer is used, it is possible to maintain operation if one of the single-phase transformers, or one winding of the three-phase unit, is damaged. This arrangement is shown in Fig. 68 and is known as the open delta or V-

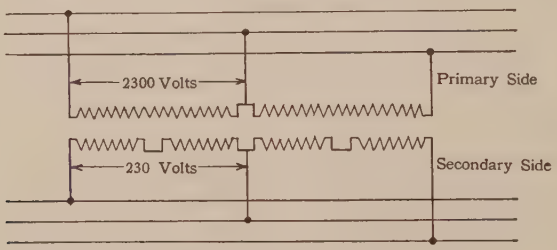


Fig. 68. Open-delta connected transformer.

connection. With the three-phase core-type design, the damaged phase cannot be isolated, due to the interlinked magnetic circuits, and the open delta connection cannot be secured. With an open delta connected three-phase shell-type transformer, the damaged phase should be short-circuited to prevent stray fluxes from the other phase from inducing voltages in the damaged windings.

T TO T CONNECTION

As with the open delta arrangement, the T to T connection requires only two single-phase transformers, as shown in Fig. 69. One of the units is called the main transformer and is provided with a 50 percent voltage tap, to which the teaser transformer is connected. The teaser unit may be designed for 86.6 of the line or main transformer

voltage, but generally is made identical with the main transformer and operated at a reduced voltage. In this way it is possible to operate two identical transformers connected T

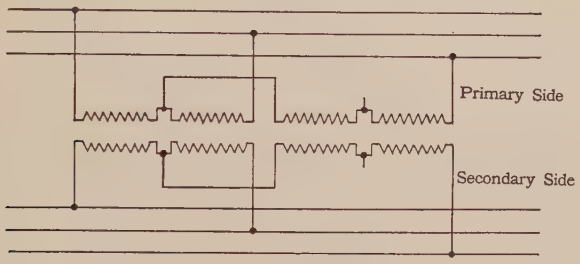


FIG. 69. T-T connected transformers.

to T, as well as open delta. Although interconnection is not required between halves of the main winding, yet each half of the primary winding must be properly wound with respect to the corresponding half of the secondary winding. Two ordinary transformers may also be used with the T to T connection, provided a 50 percent tap is available. It is also slightly more economical to operate with the T to T connection than with the V connection, when one transformer has burned out. The T to T connection can be used for operating a synchronous converter, the transformer neutral being brought out for the three-wire direct-current service. The neutral is brought out from a point at one-third the height of the teaser winding, and the ampere turns of the direct current will neutralize each other.

INTERCONNECTED STAR CONNECTION

Synchronous converters are frequently installed when a three-wire direct-current circuit is required and the T to T connection is not suitable. The three-wire arrangement is obtained by connecting the neutral wire to the neutral point of the low-voltage winding of the transformers. In such

cases the connections should be arranged so that the direct current in each transformer divides into two branches of equal ampere turns. This is to prevent a unidirectional magnetic flux in the transformer which, when superimposed on the normal magnetic cycle, would tend to raise the magnetic induction beyond saturation. This in turn would tend to cause excessive exciting current and heating, except in cases where the unbalanced current is comparatively small. Such a condition is shown in Fig. 70, which rep-

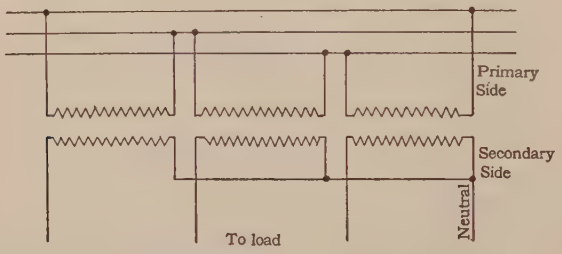


FIG. 70. Delta-star connected transformers for three-phase transformation to operate synchronous converters.

resents a delta-star connected step-down transformer with the neutral brought out. In this case each transformer low-voltage winding receives one-third of the neutral current; and if this current is not small as compared with the exciting current of the transformer, it will cause an increase in the magnetic density. The inter-connected star arrangement, as shown in Fig. 71, eliminates the flux distortion due to the unbalanced direct current in the neutral. Two separate interconnected windings are used for each leg of the star. The unbalanced neutral current flowing in this system may be compared in its action to the effect of a magnetizing current in a transformer. The effect of the main transformer currents in the high- and low-voltage windings is balanced with regard to the flux in the magnetic circuit, which depends upon the magnetizing current.

When a direct current is passed through the transformer, unless the fluxes produced by it neutralize one another, its effect on the transformer iron varies as the magnetizing current. For example, assume a transformer having a normal current capacity of 100 amperes and approximately six amperes magnetizing current, and assume that three such

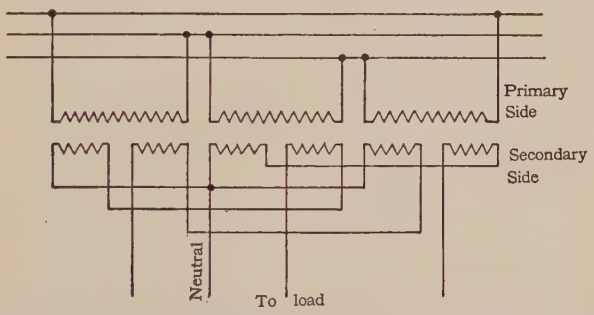


FIG. 71. Delta to interconnected star connection for three-phase transformation to operate synchronous converters.

transformers are used with star-connected, low-voltage windings for operating a synchronous converter connected to a three-wire system. Allowing 25 percent unbalancing, the current will divide equally among the three phases, giving 8.3 amperes per phase, which is more than the normal magnetizing current. The loss due to this current, however, is inappreciable, but the increased iron loss may be considerable. If a distributed winding is used, the direct current flows in opposite directions around the two halves of each winding, thus neutralizing the flux distortion.

Whether the simple star or the interconnected star connection is to be usd in a particular case, is a question of balancing the increased iron loss of the straight star connection against the increased copper loss, and the greater cost of the interconnected star arrangement. The straight star connection is much simpler, and it would be permissible

to use it for transformers of small capacities where the direct current in the neutral is not more than 30 percent (10 percent per transformer) of the rated transformer current.

With three-phase core-type transformers, it is not necessary to use the interconnected star connection, as in such transformers the direct-current flows along the magnetic circuit in the same direction in all three legs. Since the direct flux must find its return path through the air and the case outside of the magnetic circuit, its effects are practically negligible.

CHAPTER XV

THREE–PHASE TRANSFORMATION WITH SINGLE–PHASE TRANSFORMERS

THE following discussion of three-phase transformation relates to the vector relations of the voltages and currents, the capacity of transformers required with the different arrangements and the regulation of the various groups. No mention is made of the peculiar conditions introduced by certain of these connections, or their relative value under operating conditions.

THREE SINGLE-PHASE TRANSFORMERS CONNECTED IN STAR OR DELTA

In a three-phase transformation the three primary or three secondary windings of the transformers may be connected in star or in delta. With the star connection the

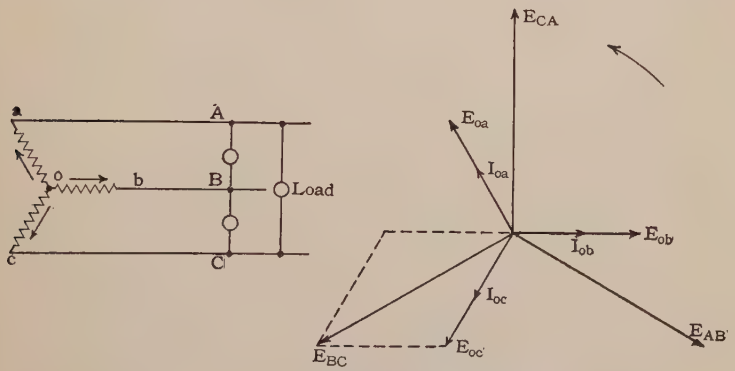


FIG. 72. Three transformers connected in star.

current in the windings is the current in the line, while the voltage of the windings is the line voltage divided by

$\sqrt{3}$. The vector relations of the various quantities are shown in Fig. 72, the load having a 100 percent power-factor. This shows that, while the current and voltage in the windings themselves are in phase, the current I_{oa} in the line aA, for example, is 30 degrees out of phase from the voltage between the lines C and A or E_{CA}. With the delta

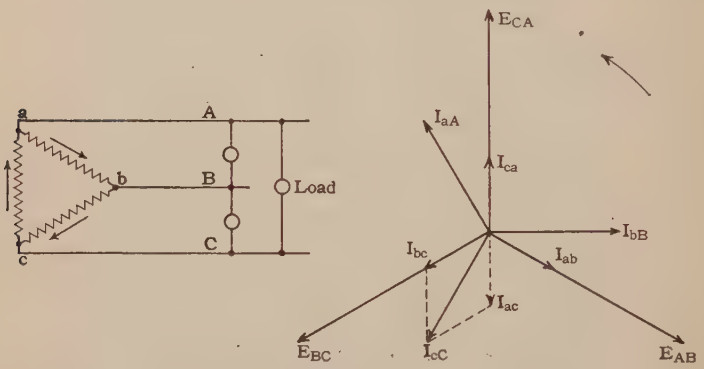


FIG. 73. Three transformers connected in delta.

connection, the voltage of the winding is the voltage of the line, and the current in each winding is the current in the line divided by $\sqrt{3}$. The vector relations of the voltages and currents are shown in Fig. 73, with a load having a 100 percent power-factor. In this case also, while the voltage and current in each of the windings are in phase, the current in the line cC for example is 30 degrees out of phase from the voltage between the lines B and C or E_{BC}. The k.v.a. transformer capacity required for a bank of star- or delta-connected transformers making a three-phase transformation is equal to the k.v.a. transformed. This is not the case with some of the other connections for the same transformation. Greater transformer capacity required in some cases is due to the fact that the currents and voltages are out of phase, with a load having a 100 percent power-

factor. The regulation of a single-phase transformer, connected in a star- or delta-connected bank, is the same as with the transformer operating single phase. With some of the other connections for the same transformation, conditions are introduced which affect the regulation of the group of transformers.

TWO TRANSFORMERS CONNECTED IN OPEN DELTA

With the group of transformers connected in delta, as shown in Fig. 73, if one of the transformers, for example *ac*, be removed, the two remaining transformers are left connected in open delta[1] as shown in Fig. 74. The voltage

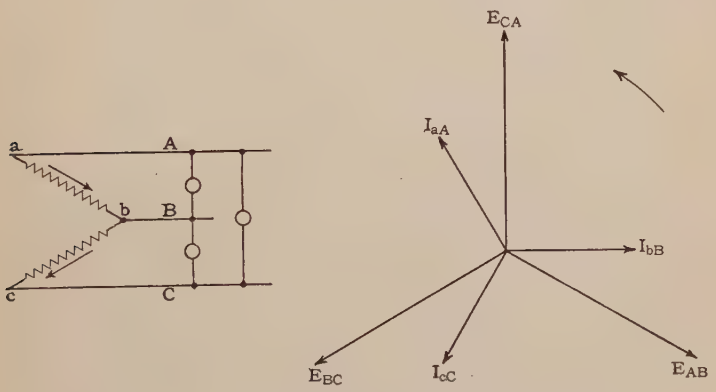


FIG. 74. Two transformers connected in open delta.

and current relations remain the same as with the delta connection and are shown in Fig. 74. The currents in the windings are now the currents in the lines, and the currents in the windings are 30 degrees out of phase from the voltages of the windings, as shown in Fig. 74. For this reason the ratio of the k.v.a. of transformer capacity required to

[1] See article "The Open Delta Connection" by Mr. J. B. Gibbs in the *Electric Journal* for Feb., 1916, p. 100.

the k.v.a. transformed is greater for this connection than with three transformers connected in delta. The k.v.a. transformed is $3EI/\sqrt{3}$, and the k.v.a. of transformer capacity required is $2EI$. Therefore,

$$\frac{\text{K.v.a. of Transformer Capacity}}{\text{K.v.a. Transformed}} = \frac{2EI}{\frac{3EI}{\sqrt{3}}} = 1.15. \quad (118)$$

In other words, with the open delta connection 15 percent greater transformer capacity is required than the k.v.a. transformed.

Assuming a balanced three-phase load on the open delta bank of transformers and that the magnitude and phase relation of the primary impressed voltages remain constant, the regulation of the different phases may be approximated as follows:

The vector relation of currents and voltages is shown in Fig. 75 for a load having a power-factor of 100 percent.

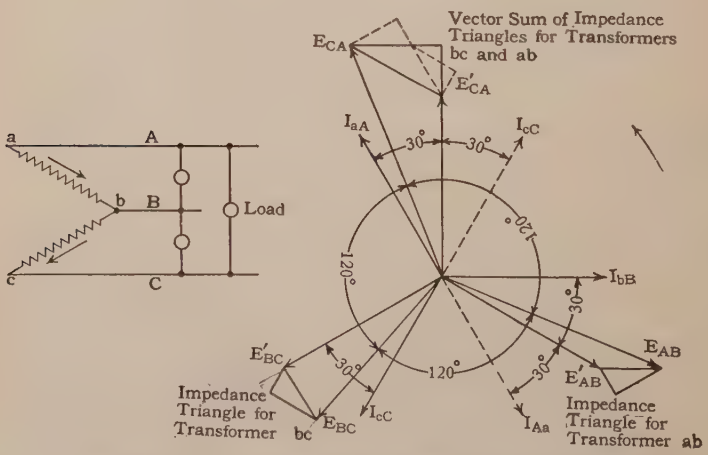


FIG. 75. Regulation of two transformers connected in open delta for load having 100 percent power-factor.

The vector E_{BC} represents the vector sum of the voltage delivered by the transformer at full load E'_{BC} and the impedance drop through the transformer as shown by the impedance triangle. The IR component of the impedance triangle is in phase with the current I_cC, and the reactive component is at right angles to this current. The current I_cC is 30 degrees in advance of the voltage E'_{BC}, and the regulation is calculated in the regular way for this condition. Therefore for the general case, from equations (97) and (98), the regulation for phase BC equals

$$IR_1 \cos(\theta - 30^\circ) \mp IX_1 \sin(\theta - 30^\circ). \qquad (119)$$

where IR_1 is the percent drop in voltage through the transformer in this phase, IX_1 is the corresponding percent reactive drop, and θ is the angle whose cosine is the power-factor of the load. The minus sign is to be used when the angle $(\theta - 30^\circ)$ is negative and the plus sign when the angle is positive; the angle θ being positive for a lagging power-factor and minus for a leading power-factor.

In a similar manner for phase AB, the IR component of the impedance triangle is in phase with the current I_{Aa}, and the reactive element is at right angles to this current. Since the current I_{Aa} is 30 degrees behind the voltage E_{AB} the regulation for phase AB equals,

$$IR_2 \cos(\theta + 30^\circ) \mp IX_2 \sin(\theta + 30^\circ). \qquad (120)$$

where IR_2 and IX_2 are the percentage resistance and reactive drops in voltage through the transformer in this phase. The minus sign is to be used when the angle $(\theta + 30^\circ)$ is negative and the plus sign when the angle $(\theta + 30^\circ)$ is positive.

The drop in voltage across the phase CA is the sum of the two impedance triangles shown in Fig. 75. Since the IR components of these triangles are in phase with the currents I_{Cc} and I_{aA}, respectively, the phase relation of the

voltage E'_{CA} and the IR component of the equivalent impedance triangle across the phase CA can be determined. Since the two components of this equivalent impedance are approximately 120 degrees apart in phase the vector sum of the resistance and reactive components are, expressed in the symbolic notation,

$$IR = 0.5\,IR_1 + IR_2 + j\,0.866\,IR_1. \qquad (121)$$

$$IX = 0.5\,IX_1 + IX_2 + j\,0.866\,IX_1. \qquad (122)$$

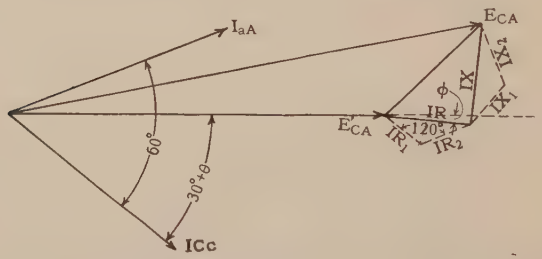


FIG. 76. Vector relations of the voltages and currents in phase CA. Reproduced to an enlarged scale.

The angle which the vector IR_1 makes with the voltage E'_{CA} in Fig. 76 is

$$\tan^{-1}\frac{0.866IR_1}{0.5IR_1 + IR_2}.$$

Therefore the angle

$$\phi = \theta + 30^\circ - \tan^{-1}\frac{0.866IR_1}{0.5IR_1 + IR_2}. \qquad (123)$$

where θ is the angle whose cosine equals the power-factor of the load. The expression may now be written for the regulation of phase CA, which equals

$$IR\cos\phi \mp IX\sin\phi. \qquad (124)$$

The minus sign is to be used when the angle ϕ is negative and the plus sign when this angle is positive.

Example 56: Find the regulation, with an 80 percent power-factor load of two 5 k.v.a. transformers connected in open delta, the full load output of the bank being 6.93 kw., or such as to load the transformers to their normal rating of 5 k.v.a. each. The percent resistance and reactive drops of the transformers under these conditions being,

For the phase BC, $IR_1 = 1.66$, $IX_1 = 2.32$

For the phase AB, $IR_2 = 2$, $IX_2 = 2.8$

The regulation of the phase BC, from equation (119),

$$= 1.66 \cos (37^\circ - 30^\circ) + 2.32 \sin (37^\circ - 30^\circ)$$
$$= 1.66 \times 0.992 + 2.32 \times 0.122 = 1.93 \text{ percent.}$$

The regulation of the phase AB from equation (120),

$$= 2 \cos (37^\circ + 30^\circ) + 2.8 \sin (37^\circ + 30^\circ)$$
$$= 2 \times 0.391 + 2.8 \times 0.92 = 3.36 \text{ percent.}$$

The regulation of the phase CA is determined as follows from equations (121) and (122),

$$IR = \sqrt{(0.5 \times 1.66 + 2)^2 + (0.866 \times 1.66)^2} = 3.17 \text{ percent.}$$

$$IX = \sqrt{(0.5 \times 2.32 + 2.8)^2 + (0.866 \times 2.32)^2} = 3.82 \text{ percent.}$$

$$\phi = 37^\circ + 30^\circ - \tan^{-1} \frac{0.866 \times 1.66}{0.5 \times 1.66 + 2} = 37^\circ + 30^\circ - \tan^{-1} 0.504 = 40^\circ$$

Then from equation (124), the regulation of phase CA,

$$= 3.17 \times 0.766 + 3.82 \times 0.643 = 4.89 \text{ percent.}$$

The regulations for loads of 100 and 80 percent power-factors may be tabulated as follows:

Phase	For Loads of 100% Power-factor	For Loads of 80% Power-factor
E'_{BC}	0.28	1.93
E'_{AB}	3.13	3.36
E'_{CA}	3.37	4.89

TWO TRANSFORMERS CONNECTED IN T

Fig. 77 shows two transformers connected in T for a three-phase transformation, and the vector relation of the voltages and currents for a load having a 100 percent

power-factor. It is evident that in the teaser transformer the current in the winding is in phase with the voltage of the winding, and in the main transformer the current is 30

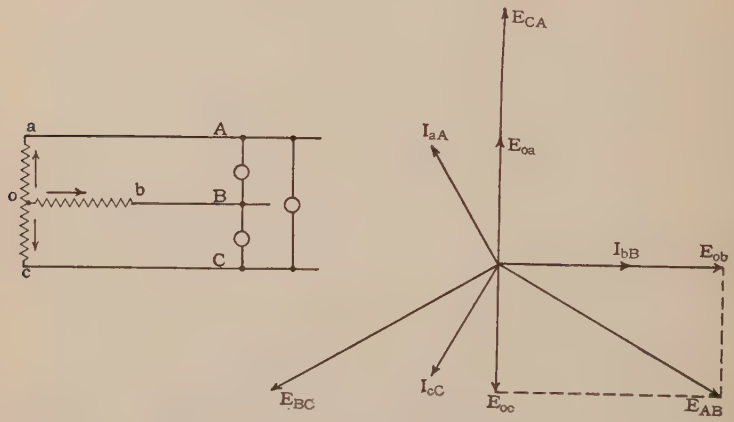


FIG. 77. Two transformers connected in T.

degrees out of phase from the voltage. The actual k.v.a. load on the transformer is therefore greater than the k.v.a. transformed. The k.v.a. transformed by the two transformers is $3EI/\sqrt{3}$, and the total transformer capacity is $EI + 0.866\,EI$. Hence,

$$\frac{\text{k.v.a. of transformer capacity}}{\text{k.v.a. transformed}} = \frac{EI + 0.866EI}{\frac{3EI}{\sqrt{3}}} = 1.075. \quad (125)$$

If the main and teaser transformers are duplicates, as is usually the case, the above becomes

$$\frac{\text{k.v.a. of transformer capacity}}{\text{k.v.a. transformed}} = \frac{2EI}{\frac{3EI}{\sqrt{3}}} = 1.15 \quad (126)$$

which is the same as for two transformers connected in open delta.

While the k.v.a. of transformer capacity required for the T connection is the same when the two units are duplicates as that for the open delta connection for the same transformation, the open delta arrangement is ordinarily preferable. With the T connection the middle point of both primary and secondary windings must be available, and the design of the main transformer must be such that the impedance encountered by the current flowing through the teaser transformer and out through the main is not high, or the regulation of the group will be seriously impaired. With standard transformers the middle points of both windings are not always available, and the two parts of the windings are not always interconnected. The question of regulation for this connection is similar in some respects to that of two Scott-connected transformers making a three-phase to two-phase transformation.

INTERCONNECTED STAR CONNECTION

The vector relations of the voltages and currents for the interconnected star connection for a load having a 100 percent power-factor are shown in Fig. 78. The voltage E_{oc}

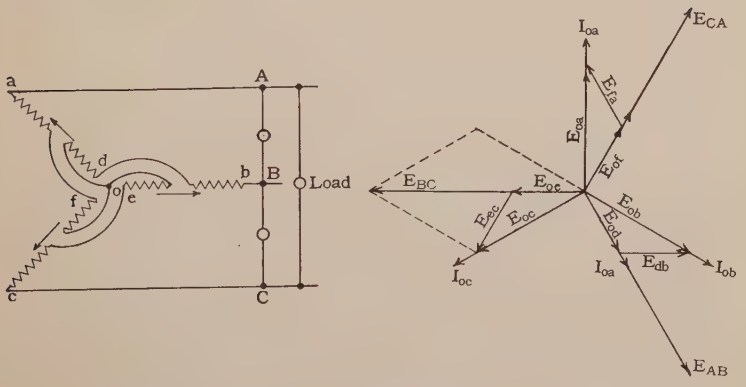


FIG. 78. Interconnected star connection of three transformers.

equals $\sqrt{3}\,E_{oe}$, and the voltage E_{BC} equals $\sqrt{3}\,E_{oc}$. Therefore E_{oe} equals one-third E_{BC}. In other words each half of the secondary winding of each transformer has a voltage of one-third of the interconnected star voltage between the lines B and C or the voltage E_{BC}. The voltage E_{oc} for example between the neutral point and one of the lines is 30 degrees out of phase with the voltages E_{oe} and E_{ec} of the two parts of secondary winding of the transformer of that phase. Therefore the k.v.a. transformed with this connection is less than the k.v.a. of transformer capacity required. The k.v.a. transformed is $\sqrt{3}\,EI$ in the primary side, whether connected delta or star (the delta connection is usually preferable) and is $6 \times \frac{1}{3}\,EI$ in the secondary side. The total k.v.a. of transformer capacity is therefore $(\sqrt{3}\,EI + 2EI)/2$ and

$$\frac{\text{k.v.a. of transformer capacity}}{\text{k.v.a. transformed}} = \frac{\frac{\sqrt{3}EI + 2EI}{2}}{\frac{3EI}{\sqrt{3}}} = 1.075. \quad (127)$$

A comparison of the various connections of single-phase transformers for a three-phase transformation as far as the relative amount of transformer capacity required is given in Table IV.

TABLE IV

SUMMARY OF THE RATIO OF THE K.V.A. OF TRANSFORMER CAPACITY TO THE K.V.A. TRANSFORMED FOR THE VARIOUS CONNECTIONS

Connection	Ratio
Three transformers—Star connected	1.0
Three transformers—Delta connected	1.0
Two transformers—Open delta connection	1.15
Two transformers—T connection	1.075
Two transformers—T connection (duplicates)	1.15
Three transformers—Interconnected star	1.075

CHAPTER XVI

PHASE TRANSFORMATION

THE most important transformer connections for phase transformations are:

1. Three-phase to single-phase.
2. Three-phase to two-phase.
3. Two-phase to six-phase.
4. Three-phase to six-phase.

This discussion does not go into detail regarding the vector relations of the voltages and currents, nor the k.v.a. capacity of transformers required for the different arrangements, but merely shows the connections required to give the transformations.

THREE-PHASE TO SINGLE-PHASE

It is impossible to deliver single-phase current from a three-phase source of supply and have balanced conditions. If this were done it would mean that the continuous flow of power from the three-phase source passing through the transformers would be changed to a flow of single-phase power passing from a maximum through zero and back to a maximum every half cycle. As this is impossible with static transformers (i.e., with any apparatus which has no capacity for storing energy at one time and subsequently giving it out), it is apparent that such a transformation is also impossible.

Various schemes have, however, been proposed for this transformation but none of them gives better results than connecting a single-phase transformer across one of the

three phases. In the case of a star-connected source of supply, this gives an equal current in two of the phases and zero current in the third. With a delta-connected source, two of the phases have the same current and the third a current twice as large as that in the other two. The current therefore has a better distribution with a delta-connected than with a star-connected source of supply.

THREE-PHASE TO TWO-PHASE

A number of schemes for three-phase to two-phase transformation have been devised, but the most commonly used is the Scott connection shown in Fig. 79, which requires two transformers. In the three-phase side, the number of turns in the teaser winding is 86.6 percent of the number in the main winding. In the two-phase side, the windings of both transformers are identical and independent, when supplying a two-phase four-wire circuit. For the sake of interchangeability the main and teaser transformers are usually made identical, that is, the three-phase windings are provided with both a 50 and an 86.6 percent tap, as shown in Fig. 79. When operating as the main transformer, the 50 percent tap is used; and when as the teaser the 86.6 percent tap, the 13.4 percent part of the winding being idle in the latter case. Each of the two halves of the three-phase winding should be distributed over the entire winding length of the magnetic circuit in order to prevent flux distortion and the resulting poor regulation.

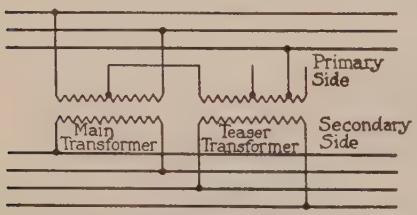


FIG. 79. Scott connection for three-phase, two-phase transformation with duplicate single-phase transformers.

A connection is sometimes used in emergency cases

where a transformer with an 86.6 percent tap is not available and a teaser transformer of the same voltage as the main transformer must be used. In this connection two transformers of exactly the same capacity and voltage are used. With this arrangement the phases are not exactly 120 degrees apart, and an attempt to operate it in parallel with a true three-phase circuit will result in unbalanced currents.

In these two Scott-connected transformations, the two-

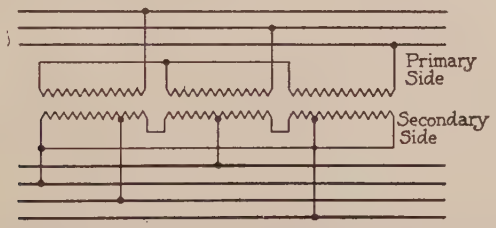


FIG. 80. Three-phase, two-phase transformation, using three single-phase transformers, with the two-phase side interconnected.

phase windings are electrically distinct. There are, however, a number of schemes in which the windings on the two-phase side are electrically interconnected, as shown in Figs. 80 and 81. In both of these transformations the three-phase side of the transformers may be connected in star or delta as desired. When a three-phase to two-phase transformation is desired in a single unit, instead of two or three transformers, either of the arrangements shown in Figs. 80 and 81 may be used with a three-phase magnetic circuit.

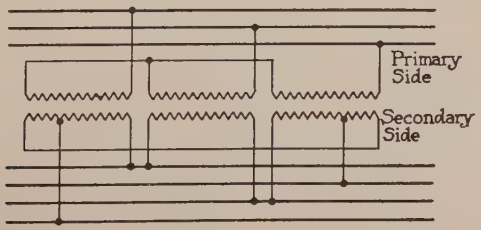


FIG. 81. Transformation from three-phase to two-phase. Three single-phase transformers with the two-phase side interconnected.

This connection is desirable when the special design of the single unit is not objectionable, as compared to the more standard single-phase transformer used for the Scott connection.

THREE-PHASE TO TWO-PHASE AND THREE-PHASE

By connecting to the corners of the delta on the secondary side of the arrangements shown in Figs. 80 and 81, three-phase current may be drawn from the transformers simultaneously with the two-phase current. The voltage however of the two-phase and three-phase circuits will be different. A connection giving the same voltage two- and three-phase on the secondary side is shown in Fig. 82.

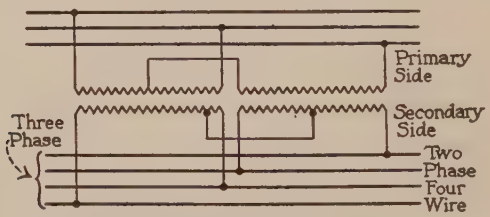


FIG. 82. Transformation from three-phase to both two-phase and three-phase with two single-phase transformers.

This is the T-connection for the three-phase transformation, and Scott connection for the three-phase to two-phase transformation.

TWO-PHASE TO SIX-PHASE

The double Scott connection shown in Fig. 83 is used where a six-phase synchronous converter is to be operated from a two-phase supply system. This transformation requires two special transformers of the same impedance, each having two low-voltage windings, connected so that their voltages are 180 degrees from each other in phase relation in order to give the six-phase system. Obviously,

considerable complication of starting taps and switches results, often to such an extent that the two units cannot be

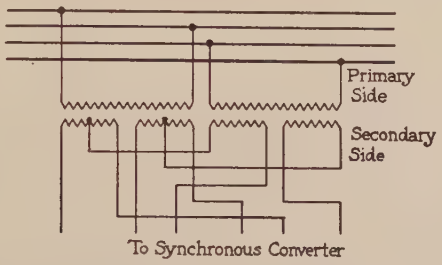


FIG. 83. Scott connection for two-phase, six-phase transformation with two single-phase transformers.

made exact duplicates of each other. In case the neutral must be brought out on the six-phase side the complications are increased.

THREE-PHASE TO SIX-PHASE

The Diametrical Connection, shown in Fig. 84, is commonly used for the three-phase to six-phase transformation.

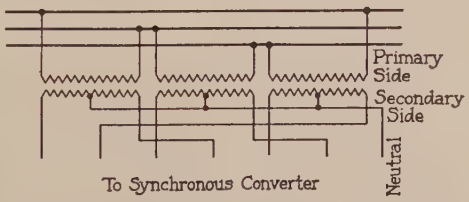


FIG. 84. Diametrical connection for transforming from three-phase to six-phase with three single-phase transformers.

It requires one low-voltage coil on each transformer which is connected to diametrically opposite points on the converter winding. It gives a simple arrangement of taps and switches for starting the converter, and continued operation at reduced capacity is possible with one transformer out of service. With diametrically connected low-voltage windings, the high-voltage windings may be connected in star or delta,

although the delta connection is usually preferable. The middle points of the diametrical windings can be connected together and brought out for the three-wire direct-current service, as the unbalanced three-wire direct current does not give a distorting effect. Arrangements should then be made for opening the neutral connections when starting, to avoid short-circuit.

With a six-phase diametrical connection with common neutral, one-half the output can be taken from the low-voltage side for operating three-phase without change of diametrical voltage. If full three-phase output should be desired, the coils can be connected in delta, in which case the diametrical voltage is increased 14 percent. The full three-phase output at 1.73 times the diametrical voltage may be obtained by connecting the coils in star, in which case the neutral should be grounded. When full output is required at the same voltage at either three-phase or six-phase, the double-delta connection is usually used.

The Double-Delta Connection requires two independent

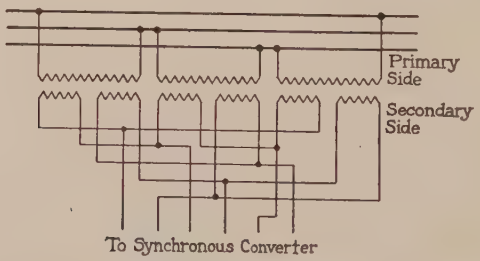


FIG. 85. Double delta connection for transforming from three-phase to six-phase with three single-phase transformers.

low-voltage windings on each transformer as in Fig. 85. Both are connected in delta, but one set is reversed as compared to the other, so that the two deltas are displaced 180 degrees in phase relation from each other. The high-voltage side should be connected in delta as this permits operation with two transformers in case the third should

become damaged. Full output three-phase may be obtained by connecting the two halves of the secondary winding of each transformer in parallel, and the three parts thus produced in delta. The double-delta connection cannot be used for three-wire direct-current service, and in this case separate autotransformers would be required to obtain the neutral point.

The Double-Star Connection, like the double-delta, requires two sets of low-voltage coils, displaced 180 degrees in phase relation from each other as in Fig. 86. The high-voltage windings may be either delta or star connected, but preferably the former.

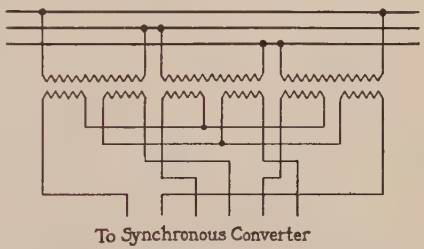


FIG. 86. Double star connection for transforming from three-phase to six-phase with three single-phase transformers.

The Double-T Connection for transforming from three-phase to six-phase is shown in Fig. 87. Like the double-

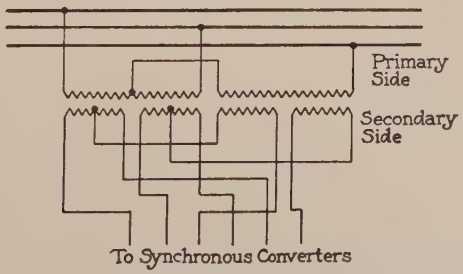


FIG. 87. Double T connection for transforming from three-phase to six-phase with two single-phase transformers.

delta and the double-star connections, two sets of low-voltage coils displaced 180 degrees in phase relation from each other are required. The two high-voltage windings are connected in T.

CHAPTER XVII

THREE–PHASE TO TWO–PHASE TRANSFORMATION WITH SINGLE–PHASE TRANSFORMERS SCOTT CONNECTED

THE Scott connection is the one most commonly used for a three-phase to two-phase transformation. Its main advantages are that practically standard transformers are used, only two units are required, and the excess of transformer capacity required over the k.v.a. transformed is not great. The main and teaser units required for the transformation can be made duplicates of each other, and thus

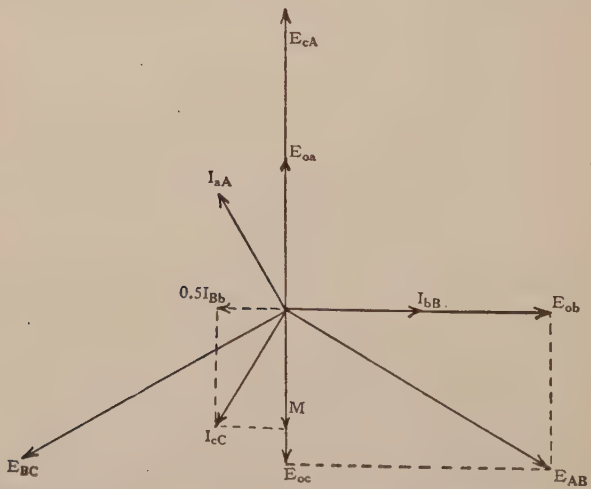


FIG. 88. Two transformers Scott connected for three-phase to two-phase transformation, showing vector relations of voltage and currents, with a ratio of voltage transformation of unity. Vector M represents the current in the three-phase winding which balances the two-phase current $I_{E'}$ in the main transformer.

interchangeable without great additional expense. The practical importance of this connection therefore warrants the following analysis as to the voltage regulation secured under load conditions.

The vector relations of the voltages and currents for a three-phase to two-phase transformation are shown in Fig. 88. With a load having a 100 percent power-factor the voltage and current in the teaser transformer are in phase, but in the main unit the voltage and current are 30 degrees out of phase. For this reason the ratio of the k.v.a. of transformer capacity required to the k.v.a. transformed is greater than unity. The current in the main transformer I_{cC} may be thought of as being made up of the two components 0.5 I_{Bb} and the current at right angles to 0.5 I_{Bb} which counterbalances the current I_{Ee} on the two-phase side of the main transformer. The current I_{aA} is also made up of two similar components.

Fig. 89 shows the two transformers connected for the three-phase to two-phase transformation. Assuming a balanced load on the transformer group, the currents in the

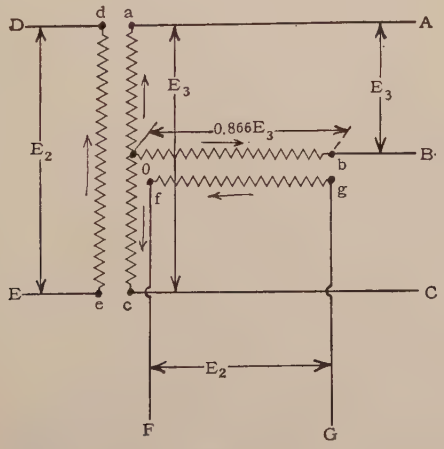


Fig. 89. Two Scott connected transformers for three-phase to two-phase transformation.

three-phase and two-phase windings are

$$I_{bB} = \frac{\text{K.v.a. transformed}}{1.73E_3}$$

and

$$I_{dD} = \frac{\text{K.v.a. transformed}}{2E_2}.$$

where E_3 is the voltage on the three-phase side and E_2 the voltage on the two-phase side. Combining these two equations gives

$$I_{bB} = \frac{2}{1.73}\frac{E_2}{E_3}I_{dD} = 1.155\frac{E_2}{E_3}I_{dD}. \qquad (128)$$

The transformer capacity required for this transformation is equal to the sum of the products of the voltage and currents for the several parts of the windings.

K.v.a. rating of three-phase side of group =
$E_3\,(I_{bB} + 0.866\ I_{bB}) = 1.866\ E_3\ I_{bB}$,

K.v.a. rating of two-phase side of group $= 2\ E_2\ I_{dD}$,

$$\text{Total K.v.a. rating of group} = \frac{1.866E_3I_{bB} + 2E_2I_{dD}}{2},$$

Substituting in this expression the value of I_{bB} from equation (128) gives

Total K.v.a. rating of group =

$$\frac{1.866E_3 1.155\frac{E_2}{E_3}I_{dD} + 2E_2I_{dD}}{2} = 2.077E_2I_{dD},$$

The k.v.a. transformed $= 2\ E_2\ I_{dD}$,

and

$$\frac{\text{Total k.v.a. rating of group}}{\text{K.v.a. transformed}} = \frac{2.077E_2I_{dD}}{2E_2I_{dD}} = 1.04. \qquad (129)$$

If the two transformers are identical, that is, interchangeable with each other, then,

$$\text{Total k.v.a. of group} = \frac{2E_3 1.155 \frac{E_2}{E_3} I_{dD} + 2E_2 I_{dD}}{2} = 2.155 E_2 I_{dD};$$

then,

$$\frac{\text{Total k.v.a. rating of group}}{\text{K.v.a. transformed}} = \frac{2.155 E_2 I_{dD}}{2E_2 I_{dD}} = 1.077. \quad (130)$$

The regulation of a Scott-connected group of transformers is a matter of some importance and it may be determined in a manner similar to that used for the two transformers connected in open delta. The impedance values necessary to determine the regulation might be measured under the actual conditions of two-phase to three-phase transformation, but it is more usual to base the calculations on the single-phase measurements made under the proper conditions on the transformers. These single-phase impedances can also be approximately calculated more easily than the three-phase impedances, if the required design data are at hand.

In the circuit *BC*, Fig. 89, there are three impedances. The first is through the teaser transformer, and is the normal impedance of the transformer measured with a single-phase current equal to I_{bB}. The other two impedances are in the main transformer. The first of these can be measured with a single-phase current equal in value to I_{bB}, and with the connection shown in Fig. 90. This measurement gives the impedance drop through each half of the main transformer with the current 0.5 I_{bB} in each half, and is the vector sum of the resistance drop in each half and the reactive element due to the magnetic separation of the two halves of the winding on the three-phase side with respect to each other. The third impedance drop is one-

half of that in the main transformer measured with a current in the two-phase side, equal to that flowing in that winding under normal operating conditions. The imped-

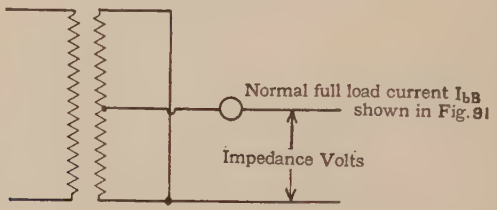


FIG. 90. Connection for determining the impedance of the main transformer in the Scott connection.

ances in the phase AB are the same as those in the phase BC. The impedance in the phase CA is that of the main transformer, measured with the normal operating current in the two-phase winding.

Let these three impedances be expressed vectorially as follows:

1. Single-phase impedance of the teaser transformer, measured with the current I_{bB} in the three-phase winding,

$$IZ_1 = IR_1 + IX_1. \qquad (131)$$

2. Single-phase impedance of the main transformer measured as shown in Fig. 90, with the current 0.5 I_{bB} in each half of the three-phase winding,

$$IZ_2 = IR_2 + IX_2. \qquad (132)$$

3. Single-phase impedance of the main transformer measured with normal operating current in the two-phase winding,

$$IZ_3 = IR_3 + IX_3. \qquad (133)$$

The impedance in phases BC and AB is $IZ_1 + IZ_2 + 0.5\ IZ_3$, and the total drop in voltage across the phase BC,

for example, is the vector sum of the three impedance triangles in this phase, shown in Fig. 91. The vector E_{BC} is the voltage of this phase at no load and E'_{BC} is the voltage

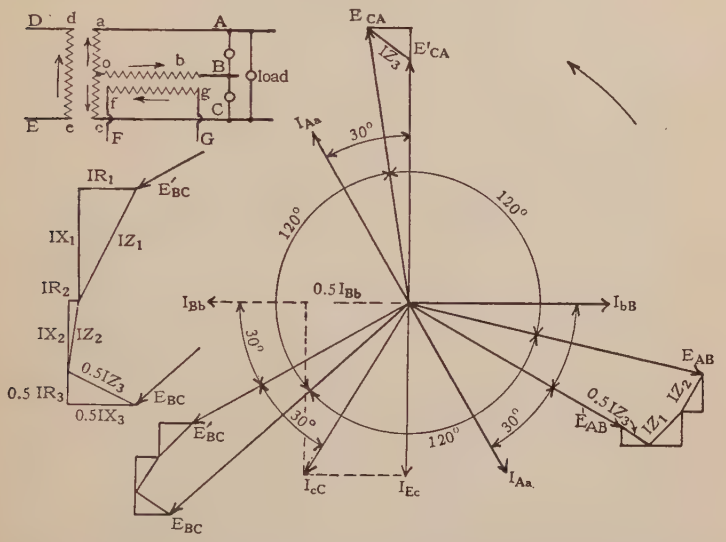


FIG. 91. Vector relations for a two-phase to three-phase transformation on the three-phase side, for a load having a 100 percent power-factor, with a ratio of voltage transformation of unity. An enlarged diagram of the vectors between E_{BC} and E'_{BC} is shown at the left drawn to scale for the values given in the example.

of the same phase at normal full load. Since the resistance drop components of the impedance triangles in phase BC are in phase with the currents I_{Bb} and I_{Ee} respectively, the phase relation of the voltage E'_{BC} and the IR component of the equivalent impedance triangle across the phase BC can be determined. In the symbolic notation, for this phase,

$$IR = IR_1 + IR_2 + j0.5\,IR_3, \qquad (134)$$

$$IX = IX_1 + IX_2 + j0.5\,IX_3. \qquad (135)$$

Therefore the regulation of phases BC and AB, by the

use of equations (97) and (98), is

$$IR\cos\phi \mp IX\sin\phi. \qquad (136)$$

The angle ϕ as shown in Fig. 92 is that between the IR component of the equivalent impedance triangle and the

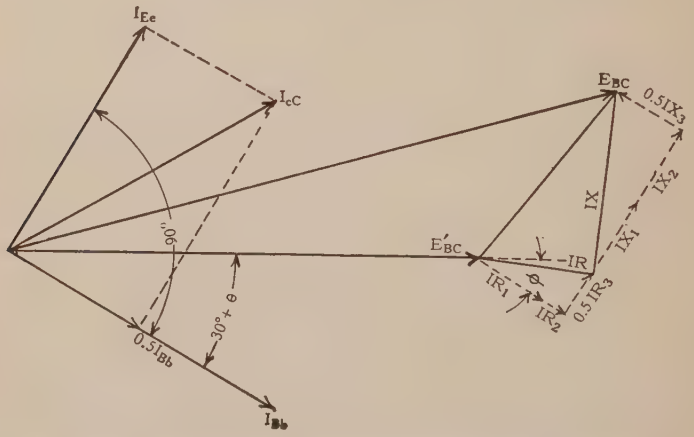


FIG. 92. Vector relations of the voltages and currents of the phase BC to an enlarged scale, for a load whose power-factor is $\cos\theta$.

voltage vector E'_{BC}. In order to determine this angle ϕ, the angle between IR_1 and IR must be known. This angle is

$$\tan^{-1}\frac{0.5IR_3}{IR_1 + IR_2}$$

and therefore

$$\phi = \theta + 30° - \tan^{-1}\frac{0.5IR_3}{IR_1 + IR_2}, \qquad (137)$$

where θ is the angle whose cosine is the power-factor of the load. In equation (136) the minus sign is used when the vector IR is ahead in phase relation of the voltage E'_{BC}, and the plus sign where IR is behind E'_{BC} in phase.

The impedance in the phase CA is IZ_3, as shown in Fig.

91. The resistance drop component of the impedance is in phase with the current I_{eE}. The regulation of this phase is

$$IR_3 \cos\theta \mp IX_3 \sin\theta, \qquad (138)$$

where θ is the angle whose cosine is equal to the power-factor of the load. The minus sign is used when the IR_3 component of the impedance drop is in advance of E'_{CA} in phase and the plus sign when it is behind E'_{CA} in phase relation.

Example 57: What is the regulation on the three-phase side of a Scott-connected group of transformers when the resistance and reactive drops under this condition are expressed in percentages as follows:

$IR_1 = 1.6$, $IX_1 = 3.2$,
$IR_2 = 0.2$, $IX_2 = 2.0$,
$IR_3 = 1.7$, $IX_3 = 3.5$,

and for a balanced load having power-factor of 80 percent?

From equations (134) and (135),

$$IR = \sqrt{(1.6 + 0.2)^2 + \left(\frac{1.7}{2}\right)^2} = 1.97 \text{ percent},$$

$$IX = \sqrt{(3.2 + 2.0)^2 + \left(\frac{3.5}{2}\right)^2} = 5.49 \text{ percent}.$$

From equation (137),

$$\phi = 37^\circ + 30^\circ - \tan^{-1}\frac{0.5 \times 1.7}{1.6 + 0.2} = 42^\circ.$$

From equation (136) the regulation is

$$1.97 \times 0.743 + 5.49 \times 0.669 = 5.14 \text{ percent}.$$

The regulation of phase *CA* from equation (138) is

$$1.7 \times 0.8 + 3.5 \times 0.6 = 3.46 \text{ percent}.$$

If the two halves of the three-phase winding on the main transformer were not properly interconnected (Fig. 94 shows the correct interconnections*), IX_2 would be greatly

* See article by the author on "Magnetic Leakage in Transformers" in the *Electric Journal* for May, 1910, p. 396, from which Fig. 94 is repeated.

increased in comparison with the value used in the above example. The calculations indicate that if IX_2 is increased, the regulation of phases BC and AB would be poorer, and also that the regulation of phase CA would not be affected.

This reasoning regarding regulation is based on the assumption of balanced three-phase voltages delivered by the transformers at no load, and a balanced three-phase load. Under these conditions the vector sum of the three load voltages at full load would be equal to zero, and consequently the vector sum of the impedance drop in the three-phases would be also equal to zero. That this is true is evident from a study of the impedance triangles in the three phases in Fig. 91.

An examination of Fig. 93 indicates that the regulation

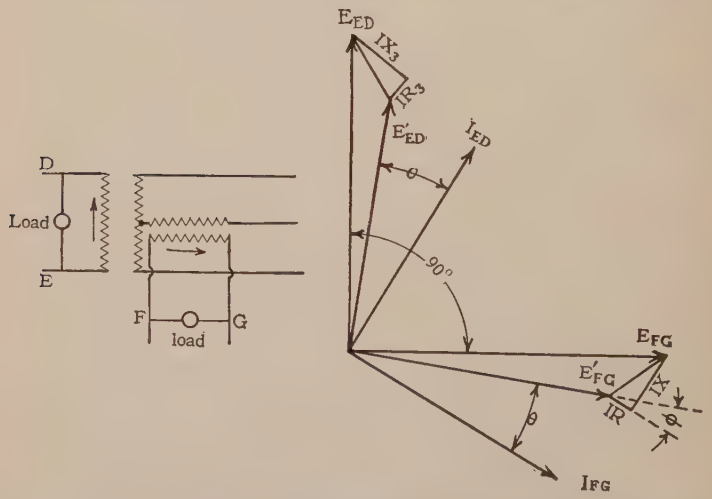


FIG. 93. Vector relations for a three-phase to two-phase transformation on the two-phase side, for a load whose power-factor is cos θ.

of the phase ED is the same as the regulation of the phase CA on the three-phase side. The impedance in the phase FG is the same as that in the phases BC and AB on the

three-phase side. Therefore, the regulation of phase FG is

$$IR \cos \phi \mp IX \sin \phi, \qquad (139)$$

when IR and IX are given by equations (134) and (135). The angle ϕ between the IR component of the equivalent impedance triangle and the voltage E'_{FG} is

$$\phi = \theta - \tan^{-1} \frac{0.5IR_3}{IR_1 + IR_2}. \qquad (140)$$

The minus sign in equation (139) is used when IR in Fig. 93 is in advance of the voltage E'_{FG} in phase relation and the plus sign when it is behind E'_{FG} in phase.

Example 58: What is the regulation on the two-phase side of a Scott connected group of transformers, when the resistance and reactive drops under this condition are

$IR_1 = 1.6$, $IX_1 = 3.2$,
$IR_2 = 0.2$, $IX_2 = 2.$,
$IR_3 = 1.7$, $IX_3 = 3.5$,

and for a balanced load having an 80 percent power-factor?

The regulation of phase ED is the same as for the phase CA on the three-phase side in the preceding example or 3.46 percent. For the phase FG the regulation is calculated as follows:

From equation (140) the angle $\phi = 37° - 25° = 12°$.

From equation (139) the regulation is

$$1.97 \times 0.978 + 5.49 \times 0.208 = 3.07 \text{ percent.}$$

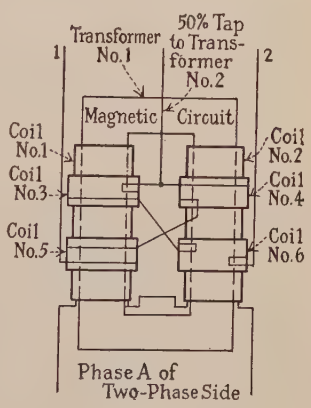


FIG. 94. Interconnections of coils on a core-type transformer for three-phase to two-phase transformation.

It is apparent that a large increase in the value of IX_2 resulting from improperly interconnected windings would not change the regulation of phase ED on the two-phase side, but would increase the regulation of the phase FG.

A summary of the regulation calculated for the two examples given is

For Three-Phase Side

(a) For phase *BC*, 5.14 percent.
(b) For phase *AB*, 5.14 percent.
(c) For phase *CA*, 3.46 percent.

For Two-Phase Side

(a) For phase *ED*, 3.46 percent.
(b) For phase *FG*, 3.07 percent.

CHAPTER XVIII

VOLTAGE TRANFORMATIONS WITH AUTO-TRANSFORMERS

An autotransformer has a single winding which must perform the functions of both the primary and secondary elements of a two-winding transformer. For this reason it is to be expected that an autotransformer may be built with less material, and that its efficiency and regulation will be superior to a two-winding transformer of the same rating. The main objection to the use of the autotransformer in a large number of cases, where it might otherwise be used to advantage, is the fact that the primary and secondary circuits are not electrically separated.

RELATION BETWEEN VOLTAGE RATIO AND OUTPUT RATING

Fig. 95 shows an autotransformer whose high voltage is E_1 and whose low voltage is E_2. The current in the high-voltage lines is I_1 and the current in the low-voltage lines is I_2. Since the current I_2, as it enters the winding at A, divides in such a ratio that the ampere-turns on each side of the tap are equal, it follows that the products of the current and voltage on each side of the tap are equal. The product of the voltage and current expressed in k.v.a. on the right of the tap in Fig. 95 is $(E_1 - E_2)\ I_1$. The output of the autotransformer is $E_1 I_1$, therefore,

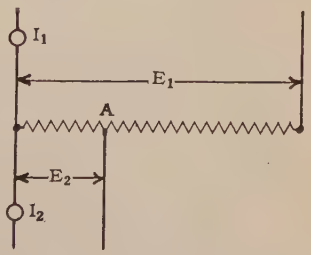


Fig. 95. Windings of an autotransformer.

$$\frac{\text{K.v.a. of transformer parts}}{\text{K.v.a. transformed}} = \frac{E_1 - E_2}{E_1} = 1 - \frac{E_2}{E_1}. \quad (141)$$

Since the ratio of voltage transformation R of the autotransformer is E_1/E_2, equation (1) may be put into the form

$$\frac{\text{K.v.a. of transformer parts}}{\text{K.v.a. transformed}} = 1 - \frac{1}{R}. \quad (142)$$

Example 59: If a transformer which has a voltage ratio of 2,300 to 460 be connected as an autotransformer, what should the normal rating of the transformer be so as to give a 12 k.v.a. output as an autotransformer?

From equation (142),

$$\text{K.v.a.} = 12\left(1 - \frac{1}{\frac{2760}{460}}\right) = 10.$$

Fig. 96 shows the rating of the transformer parts which

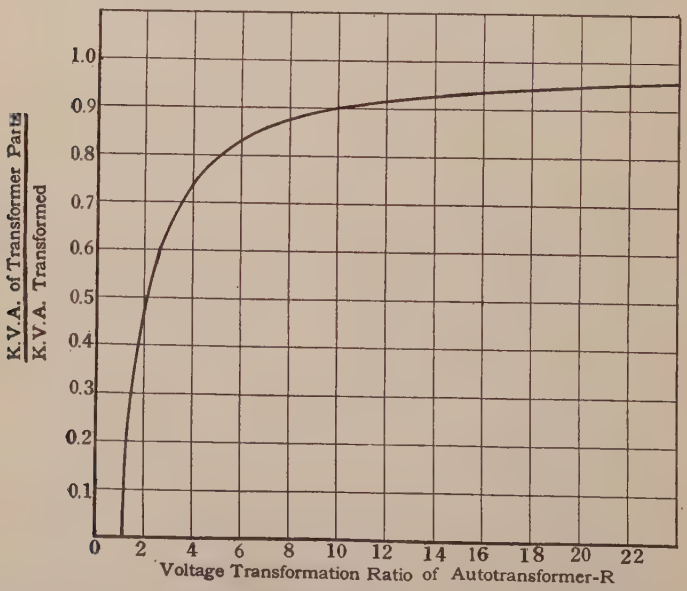


FIG. 96. Effect of voltage transformation ratio of an autotransformer on the ratio of the k.v.a. of transformer parts to the k.v.a. transformed.

are required for a given transformation, as a percentage of the k.v.a. transformed, for various voltage ratios of transformation. It is apparent that the rating of the transformer parts increases rapidly with increasing ratios of transformation up to 5, and, for ratios greater than 10, is practically constant.

Since a two-winding transformer can be reconnected as an autotransformer and give an increased rating, it is evident that, with the same rating, the autotransformer could be built with less material. When the transformer windings are reconnected to form an autotransformer with a greater ouput rating, the current densities and consequently the losses remain the same and, since the output has increased, the efficiency is increased. Since the percentage copper loss is less, the regulation at high power-factors is evidently improved. It now remains to show the effect on the percentage impedance of the transformer when it is reconnected to form an autotransformer.

RELATION OF TRANSFORMER TO AUTOTRANSFORMER IMPEDANCE

The impedance of a two-winding transformer may be determined by short-circuiting the high-voltage winding and impressing voltage on the low-voltage winding; or by short-circuiting the low-voltage winding and impressing voltage on the high-voltage winding. If E be the voltage required to circulate full-load currents in the windings in either case, and E_n be the normal voltage of the winding upon which the voltage E is impressed, by definition,

$$Z_T = \frac{E}{E_n}.$$

If I_n be the normal current in the circuit upon which voltage is impressed, this equation may be written

$$Z_T = \frac{EI_n}{E_n I_n}.$$

In other words,

$$Z_T = \frac{\text{Volt-amperes taken on short-circuit}}{\text{Rated volt-amperes of transformer}}. \qquad (143)$$

Similarly, the impedance of an autotransformer is

$$Z_A = \frac{\text{Volt-amperes taken on short-circuit}}{\text{Rated volt-amperes of autotransformer}}. \qquad (144)$$

If EI_n be the volt-amperes taken on short-circuit by a two-winding transformer connected by either of the methods

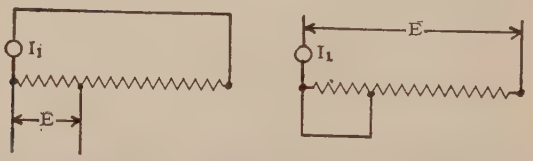


FIG. 97. Connections for measuring the impedance volts of an autotransformer.

shown in Fig. 97, and E_nI_n is the normal output rating of the transformer, the impedance as shown by equation (144) is

$$Z_A = \frac{EI_n}{(E_nI_n)\dfrac{R}{R-1}}. \qquad (145)$$

The output rating of the autotransformer being taken from equation (142) as being $(R/(R-1))$ times the rating of the windings connected as a transformer,

$$\text{Or } Z_A = \frac{E}{E_n}\left(\frac{R-1}{R}\right).$$

Now E/E_n is the impedance of the windings connected as a transformer, therefore,

$$Z_A = Z_T\left(1 - \frac{1}{R}\right). \qquad (146)$$

It is evident from the preceding that the lower impedance of the windings connected as an autotransformer is due to the greater output rating of the autotransformer as compared to the output rating of the windings connected as a transformer. The impedance of an autotransformer may be measured directly by either of the connections shown in Fig. 97, and the above demonstration is given only to show the relation between the impedance of the two windings connected as a transformer and as an autotransformer.

Example 60: If a 10 k.v.a. transformer having voltages of 2,300 to 460 volts has an impedance of three percent, its percent impedance connected as a 12 k.v.a. autotransformer will be

$$Z_A = 3\left(1 - \frac{1}{\frac{2760}{460}}\right) = 3 \times 0.83 = 2.5 \text{ percent.}$$

Since both the percentage copper loss and reactance is less for the windings connected as an autotransformer, the regulation will be better at all power-factors.

THREE SINGLE-PHASE AUTOTRANSFORMERS

Three single-phase autotransformers may be connected in star as shown in Fig. 98 to secure a three-phase transforma-

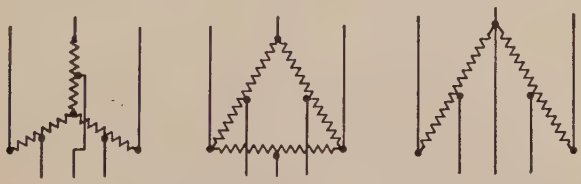


FIG. 98. Star connection.
FIG. 99. Delta connection.
FIG. 100. Open delta connection.

tion. The k.v.a. of transformer capacity required depends on the ratio of voltage transformation, and bears the same ratio to the k.v.a. transformed as do the same quantities for a single autotransformer performing a single-phase trans-

formation. While a three-phase voltage transformation may be secured by the connections shown in Fig. 99, this arrangement is not practicable except for special cases, and for small ratios of voltage transformation, because of its inefficiency. For example, with a voltage ratio of 2 to 1 as shown in Fig. 99, the output would only be 58 percent of that for the star connection shown in Fig. 98 with the same k.v.a. of transformer capacity. The low voltage of a single-phase autotransformer, or of the star-connected group shown in Fig. 98, is in phase with the high voltage. But with the connection shown in Fig. 99 there is a phase rotation of the delivered voltage as compared to that impressed. This is a further objection to this connection, as parallel operation would not be possible with the star connection shown in Fig. 98.

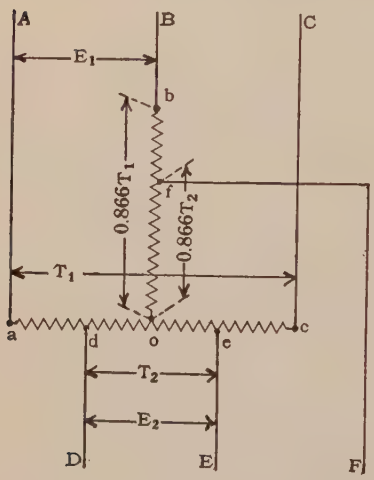


FIG. 101. T-connection.

TWO-SINGLE PHASE AUTOTRANSFORMERS CONNECTED IN OPEN DELTA

Two single-phase autotransformers may be connected in open delta as shown in Fig. 100, to secure a three-phase transformation. The k.v.a. of transformer capacity required for the transformation depends on the ratio of voltage transformation, and requires approximately 15 percent more capacity than a single-phase autotransformer, making the same voltage transformation single-phase. The additional capacity required is the same as that required by

two single-phase transformers operating open delta. Aside from the question of capacity the open-delta connection has an advantage as compared to two T-connected autotransformers in that the units are duplicates of each other, and there is no possibility of bad regulation as with the T connection if the two halves of the main autotransformers are not properly interconnected.

T-CONNECTED AUTOTRANSFORMERS

Two transformers are shown in Fig. 101 T-connected for a three-phase to three-phase transformation. The k.v.a. of the teaser is the sum of the products of the voltage and currents in the two parts or

$$\text{K.v.a.} = 0.866\, E_2 I_{fo} + (0.866\, E_1 - 0.866\, E_2)\, I_{bB}; \quad (147)$$

but

$$I_{fo} = I_{Ff} + I_{Bb}. \quad (148)$$

Also, since the currents on the high- and low-voltage sides of the group are inversely as the voltages

$$I_{Ff} = \frac{E_1}{E_2} I_{bB}. \quad (149)$$

Substituting this value of I_{Ff} in equation (148) gives

$$I_{fo} = \left(\frac{E_1}{E_2} - 1\right) I_{bB}.$$

Substituting this value of I_{fo} in turn in equation (147) gives

$$\text{K.v.a.} = 1.73\, I_{bB}\, (E_1 - E_2). \quad (150)$$

Similarly the k.v.a. of the main autotransformer is

$$\text{K.v.a.} = E_2 I_{do} + (E_1 - E_2)\, I_{bB}. \quad (151)$$

Substituting the value of I_{do} in terms of I_{bB}, which is a similar relation to that expressed in equation (149) gives

$$\text{K.v.a.} = 2\, (E_1 - E_2)\, I_{bB}. \quad (152)$$

The k.v.a. transformed is

$$\text{K.v.a.} = 1.73\, I_{bB}\, E_1. \tag{153}$$

Now from equations (150) and (153),

$$\frac{\text{K.v.a. of teaser}}{\text{K.v.a. transformed}} = \frac{1.73 I_{bB}(E_1 - E_2)}{1.73 I_{bB} E_1} = 1 - \frac{E_2}{E_1}. \tag{154}$$

Also from equations (10) and (12),

$$\frac{\text{K.v.a. of main unit}}{\text{K.v.a. transformed}} = \frac{2(E_1 - E_2) I_{bB}}{1.73 I_{bB} E_1} = 1.15\left(1 - \frac{E_2}{E_1}\right) \tag{155}$$

and,

$$\frac{\text{K.v.a. of both units}}{\text{K.v.a. transformed}} = 1 - \frac{E_2}{E_1} + 1.15\left(1 - \frac{E_2}{E_1}\right)$$

$$= 2.15\left(1 - \frac{E_2}{E_1}\right). \tag{156}$$

If the ratio of transformation is $R = (E_1/E_2)$, equation (156) may be written

$$\frac{\text{K.v.a. of both units}}{\text{K.v.a. transformed}} = 2.15\left(1 - \frac{1}{R}\right). \tag{157}$$

This expression gives the total k.v.a. of the autotransformers, and to put it on the same basis as for a two-winding transformer, the expression must be divided by two. The k.v.a. of a two-winding transformer is not the product of the current and voltages for both primary and secondary windings, but the product for one winding only.

Therefore, equation (157) becomes

$$\frac{\text{K.v.a. of both units}}{\text{K.v.a. transformed}} = 1.075\left(1 - \frac{1}{R}\right). \tag{158}$$

As is to be expected this expression is the same as for

two T-connected transformers except for the addition of the term $[1 - (1/R)]$, which takes care of the autotransformer feature.

TABLE V

RATIO OF THE K.V.A. OF TRANSFORMER CAPACITY TO THE K.V.A. TRANSFORMED FOR THE VARIOUS THREE-PHASE CONNECTIONS

Connection	Ratio
Three autotransformers star connected	$1.0\left(1-\frac{1}{R}\right)$
Three autotransformers delta connected	greater than $\left(1-\frac{1}{R}\right)$
Two autotransformers, open-delta connected	$1.15\left(1-\frac{1}{R}\right)$
Two autotransformers, T connected	$1.075\left(1-\frac{1}{R}\right)$

CHAPTER XIX

PHASE TRANFORMATION WITH AUTOTRANSFORMERS

WITH two-winding transformers the Scott connection is the most desirable arrangement for securing a three-phase to two-phase transformation, and with autotransformers the connection still retains its advantages. There are three cases to be considered when analyzing this transformation for autotransformers.

I. WHEN THE TWO-PHASE VOLTAGE IS GREATER THAN 86.6 PERCENT OF THE THREE-PHASE VOLTAGE

Fig. 102 shows two autotransformers, connected for this transformation. A condition of balanced load is assumed in order to simplify the problem. In determining the k.v.a. rating of the autotransformers, it is necessary to know the currents in the various parts of the windings. The currents whose values are not obvious are I_{fd} and I_{bf}. From Fig. 102,

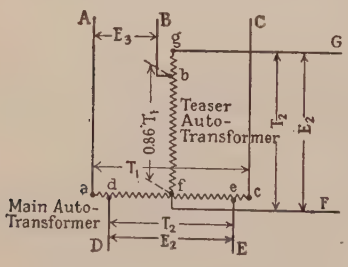


FIG. 102. Connections for the case where the two-phase voltage is greater than 86.6 percent of the three-phase voltage.

$$I_{fd} = I_{dD} + I_{aA},$$

The phase relations of these currents are shown in Fig. 103, and the numerical value

$$I_{fd} = -I_{dD} + I_{aA} (\cos 30° + j \sin 30°).$$

Since the cosine of 30 degrees is 0.866 and the sine of

30 degrees is 0.5,

$$I_{fd} = -I_{dD} + 0.866\, I_{aA} + j\, 0.5\, I_{aA}. \qquad (159)$$

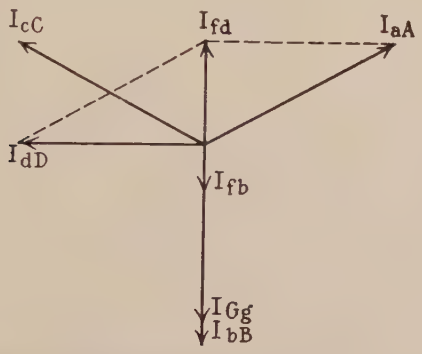


FIG. 103. Phase relations for the connec- tions shown in Fig. 102.

The next step is to secure an expression for I_{aA} in terms of I_{dD},

$$I_{aA} = \frac{\text{K.v.a. transformed}}{1.73 E_3}$$

and

$$I_{dD} = \frac{\text{K.v.a. transformed}}{2E_2}.$$

Combining these two equations gives

$$I_{aA} = \frac{2}{1.73}\frac{E_2}{E_3} I_{dD}. \qquad (160)$$

Substituting this value of I_{aA} in equation (159) gives

$$I_{fd} = \left(\frac{E_2}{E_3} - 1 + j0.577\frac{E_2}{E_3}\right) I_{dD}$$

$$= \sqrt{\left(\frac{E_2}{E_3} - 1\right)^2 + 0.333\left(\frac{E_2}{E_3}\right)^2}\, I_{dD}. \qquad (161)$$

The current in the remaining part of the main autotrans-

former is aA, whose value, in terms of I_{dD}, is given in equation (160). The sum of the products of the voltage and current in the two parts of the main autotransformer winding gives its k.v.a. rating as follows:
K.v.a. of main auto =

$$\left[\sqrt{\left(\frac{E_2}{E_3}-1\right)^2+0.333\left(\frac{E_2}{E_3}\right)^2}E_2+1.154\frac{E_2}{E_3}(E_3-E_2)\right]I_{dD}. \quad (162)$$

The total k.v.a. transformed =

$$2E_2I_{dD}. \quad (163)$$

Then,

$$\frac{\text{K.v.a. of main auto}}{\text{K.v.a. transformed}} =$$

$$\frac{1}{2}\left\{\sqrt{\left(\frac{E_2}{E_3}-1\right)^2+0.333\left(\frac{E_2}{E_3}\right)^2}+1.154\left(1-\frac{E_2}{E_3}\right)\right\}. \quad (164)$$

From Fig. 102

$$I_{bf}=\left(\frac{T_2}{0.866T_1}-1\right)I_{dD},$$

and since

$$\frac{E_3}{E_2}=\frac{T_1}{T_2},$$

$$I_{bf}=\left(\frac{1}{0.866}\frac{E_2}{E_3}-1\right)I_{dD}.$$

Therefore,

K.v.a. of teaser auto $= 2\ (E_2 - 0.866\ E_3)\ I_{dD}$

and

$$\frac{\text{K.v.a. of teaser auto}}{\text{K.v.a. transformed}}=1-\frac{0.866}{\frac{E_2}{E_3}}. \quad (165)$$

Equations (164) and (165) give the total k.v.a. rating

of the autotransformers, and to put them on the same basis as for a two-winding transformer, the expressions must be divided by two. Therefore, equations (164) and (165) become

$$\frac{\text{K.v.a. of transformer parts required for main auto}}{\text{K.v.a. transformed}} = \frac{1}{4}\left\{\sqrt{\left(\frac{E_2}{E_3}-1\right)^2+0.333\left(\frac{E_2}{E_3}\right)^2}+1.154\left(1-\frac{E_2}{E_3}\right)\right\}, \quad (166)$$

$$\frac{\text{K.v.a. of transformer parts required for teaser auto}}{\text{K.v.a. transformed}} = \frac{1}{2}\left\{1-\frac{0.866}{\frac{E_2}{E_3}}\right\}. \quad (167)$$

Example 61: What is the ratio for the main and teaser autotransformers of the k.v.a. rating of the transformer parts required to the k.v.a. transformed, for a ratio of transformation of 2,300 volts three-phase to 2,300 volts two-phase?

For this case, $\frac{E_2}{E_3} = 1$, and from equation (166),

$$\frac{\text{K.v.a. of transformer parts required for main auto}}{\text{K.v.a. transformed}} = \tfrac{1}{4}[\sqrt{(1-1)^2+0.333}+1.154(1-1)] = 0.144.$$

Also, from equation (167),

$$\frac{\text{K.v.a. of transformer parts required for teaser auto}}{\text{K.v.a. transformed}} = \tfrac{1}{2}(1-0.866) = 0.067.$$

2. WHEN THE TWO-PHASE VOLTAGE IS LESS THAN 86.6 PERCENT OF THE THREE-PHASE VOLTAGE

Fig. 104 shows the two autotransformers connected for this condition. Evidently, the conditions relating to the main autotransformer are the same in this case as in the preceding one, but are different for the teaser. The current I_{fg} is determined as follows:

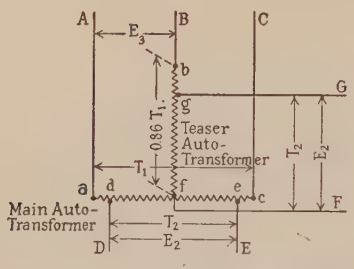


FIG. 104. Connections for the case where the two-phase voltage is less than 86.6 percent of the three-phase voltage.

$$T_2 I_{fg} = (0.866\,T_1 - T_2)\,I_{aA},$$

$$I_{fg} = \left(\frac{0.866\,T_1}{T_2} - 1\right) I_{aA}.$$

The k.v.a. of teaser auto-transformer =

$$\left[\left(\frac{0.866 E_3}{E_2} - 1\right) E_2 + (0.866 E_3 - E_2)\right] I_{aA}.$$

Substituting the value of I_{aA} from equation (103) gives

$$\text{K.v.a. of teaser auto} = 2\left(1 - 1.154\frac{E_2}{E_3}\right) E_2 I_{dD}. \quad (168)$$

By the use of equation (163),

$$\frac{\text{K.v.a. of teaser auto}}{\text{K.v.a. transformed}} = 1 - 1.154\frac{E_2}{E_3} \quad (169)$$

Putting this relation on the same basis as for a two-winding transformer it becomes

$$\frac{\text{K.v.a. of transformer parts required for teaser auto}}{\text{K.v.a. transformed}} = \frac{1}{2}\left(1 - 1.154\frac{E_2}{E_3}\right). \quad (170)$$

Example 62: What is the ratio for the main and teaser autotransformers of the k.v.a. rating of the transformer parts required to the k.v.a. transformed, for a ratio of transformation of 2,200 volts three-phase to 1,800 volts two-phase?

For this case $E_2/E_3 = 0.78$ and from equation (166),

$$\frac{\text{K.v.a. of transformer parts required for main auto}}{\text{K.v.a. transformed}} = \tfrac{1}{4}[\sqrt{(0.78 - 1)^2 + 0.333(0.78)^2} + 1.154(1 - 0.78)] = 0.188,$$

and from equation (170),

$$\frac{\text{K.v.a. of transformer parts required for teaser auto}}{\text{K.v.a. transformed}} = \tfrac{1}{2}[1 - 1.154 \times 0.78] = 0.05.$$

3. WHEN THE TWO-PHASE VOLTAGE IS GREATER THAN THE THREE-PHASE VOLTAGE

An analysis of this case, the arrangement of which is given in Fig. 105, shows that the current I_{fa} is the same as the current I_{fa} in equation (161). The current in the outer ends of the main autotransformer is the two-phase current I_{aD}. The k.v.a. of the main autotransformer is, therefore, K.v.a. of main auto =

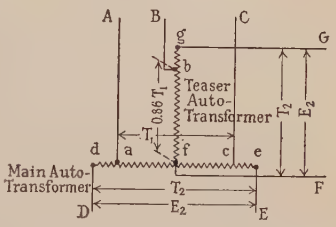


FIG. 105. Connections for the case where the two-phase voltage is greater than the three-phase voltage.

$$\left\{\sqrt{\left(\frac{E_2}{E_3} - 1\right)^2 + 0.333\left(\frac{E_2}{E_3}\right)^2}\, E_3 + (E_2 - E_3)\right\} I_{aD} \quad (171)$$

or

$$\frac{\text{K.v.a. of main auto}}{\text{K.v.a. transformed}} = \frac{1}{2}\left\{\sqrt{\left[1 - \frac{1}{\frac{E_2}{E_3}}\right]^2 + 0.333} + \left[1 - \frac{1}{\frac{E_2}{E_3}}\right]\right\}. \quad (172)$$

Putting this relation on the same basis as for a two-winding transformer it becomes

$$\frac{\text{K.v.a. of transformer parts required for main auto}}{\text{K.v.a. transformed}} = \frac{1}{4}\left\{\sqrt{\left[1 - \frac{1}{\frac{E_2}{E_3}}\right]^2 + 0.333} + \left[1 - \frac{1}{\frac{E_2}{E_3}}\right]\right\}. \quad (173)$$

For this case the relation of the k.v.a. of the teaser autotransformer to the k.v.a. transformed is the same as that shown in equation (167).

Example 63: What is the ratio for the main and teaser autotransformer of the k.v.a. rating of the transformer parts required to the k.v.a. transformed, for a ratio of transformation of 2,300 volts three-phase to 2,500 volts two-phase?

For this case, $\frac{E_2}{E_3} = 1.09$, and for the main autotransformer, from equation (173),

$$\frac{\text{K.v.a. of transformer parts required for main auto}}{\text{K.v.a. transformed}} = \frac{1}{4}\left\{\sqrt{\left(1 - \frac{1}{1.09}\right)^2 + 0.333} + \left(1 - \frac{1}{1.09}\right)\right\} = 0.165$$

and from equation (167),

$$\frac{\text{K.v.a. of transformer parts required for teaser auto}}{\text{K.v.a. transformed}} = \frac{1}{2}\left(1 - \frac{0.866}{1.09}\right) = 0.102.$$

Fig. 106 shows the variation of the k.v.a. of transformer

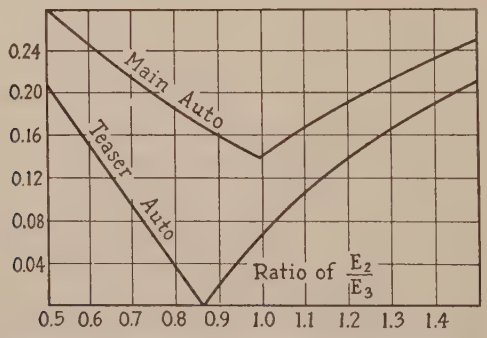


FIG. 106. Effect of voltage ratio on transformer capacity required. The ordinates represent the ratio of k.v.a. of transformer parts required to the k.v.a. transformed.

parts required to the k.v.a. transformed for both the main and teaser autotransformers, for ratio of transformations E_2/E_3, ranging from 0.5 to 1.5.

CHAPTER XX

POLARITY

If two transformers are connected in parallel and a large exchange current results, the position of either the primary or secondary terminals of one of them must be interchanged. This will reverse the time-phase relation of the secondary voltage of that particular unit, and the condition for parallel operation will be established. In connecting transformers in parallel, if the relative direction of the primary and secondary windings around the magnetic circuit, and the position of the leads issuing from the case are the same on the two units, similarly located leads should be connected together. Transformers which may be connected for parallel operation, with similarly located leads joined, are said to have the same polarity. If leads which are not similarly located are required to be connected together in order to obtain parallel operation, the transformers are said to have opposite polarity. Transformers of a given type have the same polarity, a standard polarity being necessary so that the transformers may be connected always in the same manner when they are to be operated in parallel. Transformers of the same class, for example distribution transformers, usually have the same polarity.

POLARITY OF SINGLE-PHASE TRANSFORMERS

If a transformer be connected as shown in Fig. 107, the value indicated by the voltmeter will be the sum or the difference of the voltages of the supply and load circuits. Whether the voltmeter will indicate the numerical sum or difference of these voltages will depend upon the relative

directions of the two windings around the magnetic circuit, and on the way the leads are brought out. Two transformers have the same polarity if the voltages indicated by a voltmeter connected on first one and then the other, as shown in Fig. 107, are the sum of the supply and load circuits for both units or are, in both cases, the difference of these voltages.

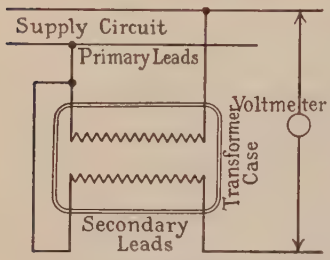


FIG. 107. Connections for determining the polarity of a single-phase transformer.

In order that the polarity of transformers shall be at once apparent so that parallel connections will be expedited, it is customary to mark the leads so that the phase relation of the voltages in the primary and secondary windings will be indicated. For example, if the leads are marked as shown in Fig. 108, the convention is to indicate that if the winding whose leads are marked 1-2 be excited by a voltage whose phase relation is indicated by the arrow pointing from 1 to 2, the phase relation of the voltage in the secondary winding will be indicated by the arrow pointing from 3 to 4. In other words, the two voltages are 180 degrees apart in time-phase relation, and consequently when leads 2 and 3 are connected together, the voltage between leads 1 and 4 would be the sum of the impressed and delivered voltages or the polarity would be additive. To operate transformers thus marked in parallel, as far as the matter of polarity is concerned, it is only necessary to connect similarly marked leads together.

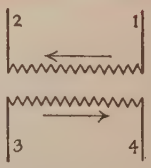


FIG. 108. Method of marking the leads to indicate polarity.

The above, while having a practical value, is a rather

superficial discussion of the subject of polarity as is evidenced by the following. Fig. 109 indicates the coils shown in Figs. 107 and 108, wound on the magnetic circuit in a continuous spiral, so as to give an additive polarity when they are connected as shown in Fig. 107. The voltage between 4 and 1 is not actually the sum of that impressed on the primary winding and that induced in the secondary winding, but is the induced voltage in the two coils connected together and considered as one winding. The arrows on the leads indicate the instantaneous directions of the currents in the windings which would result if 1 and 4 were connected together. If the coil 3–4 had been wound around the magnetic circuit in the direction opposite to that of 1-2, the induced voltages in the two coils would have opposed each other and the polarity of the transformer would have been subtractive.

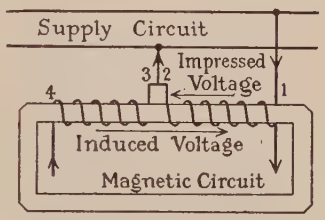


FIG. 109. Relative direction of the impressed and induced voltages in the windings of a single-phase transformer whose polarity is additive when the leads are connected as indicated in Fig. 107.

POLARITY OF THREE-PHASE TRANSFORMERS

It has been shown that, if two similar single-phase transformers be connected in parallel, and a condition results which is not operative, it is only necessary to reverse either the primary or secondary connections of one transformer and the difficulty will disappear. With three-phase transformers the problem is not so simple, and the different methods by which the coils are wound or connected may result in conditions in which it is not possible to parallel one transformer with another by interchange of the leads

on the outside of the case.[1] This will be seen by comparing the several connections with the corresponding vector diagram of voltages.

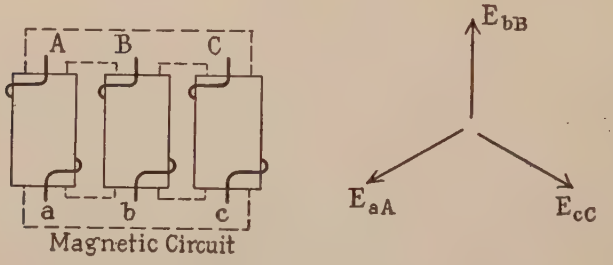


FIG. 110. The assumed directions of the voltages in the three coils when a three-phase flux exists in the magnetic circuit.

Let the direction of winding the coils, for example, be indicated by Fig. 110 which shows that if the current enters by lead *A*, it goes left handedly around the coil, and if by lead *a*, it goes around right handedly. The change from left-handed to right-handed circulation of the current around the magnetic circuit could also be changed by actually changing the direction of winding the coil. Assume that a three-phase flux exists in the magnetic circuit and let the voltages of the three coils be represented by the arrows in Fig. 110. If these three

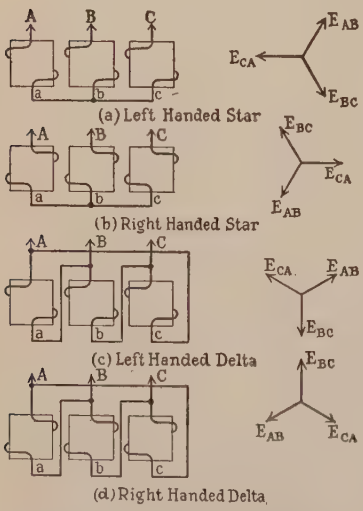


FIG. 111. The phase relation of the three-phase voltages resulting from the several connections shown.

[1] See article on "Polarity of Transformers for Parallel Operation" by W. M. McConahey in the *Electric Journal* for July, 1912, p. 613; and on "Polarity of Transformers" by W. M. Dann, in the *Electric Journal* for July, 1916, p. 350.

coils are connected in the several ways shown in Fig. 111, the three-phase voltages resulting are shown by the vector diagrams. The left-handed star connection, for example, is so called because current entering on lead *A* passes left handedly around the coil. Fig. 111 shows the four different connections, which give voltages differing in phase relation.

The primary windings of a given transformer may be connected to give voltages shown in *a, b, c,* or *d,* and the secondaries may be connected in a similar manner. This makes possible 16 different connections on one transformer, and the problem is to determine how many of these combinations will allow one transformer to be paralleled with another transformer having the same possible connections. When comparing a certain connection on the primary with one on the secondary, the phase relation of the voltages, as shown by *a, b, c, d,* must be rotated 180 degrees to represent the true secondary condition. The reason for this being that with similar windings on primary and secondary, the secondary voltage of a transformer is 180 degrees out of phase

Combinations		Angular Displaeemcnt, Degrees
Primary	Secondary	
a	*a*	180
a	*b*	0
a	*c*	30 advance
a	*d*	30 retardation
b	*a*	0
b	*b*	180
b	*c*	30 retardation
b	*d*	30 advance
c	*a*	30 retardation
c	*b*	30 advance
c	*c*	180
c	*d*	0
d	*a*	30 advance
d	*b*	30 retardation
d	*c*	0
d	*d*	180

from that impressed on the primary side. The following table gives the angular displacement between the primary impressed and secondary delivered voltages for all of the possible combinations, Fig. 111.

The combinations will operate in parallel in which the primary and secondary voltages are out of time-phase by the same amount, except those cases where the displacement is in advance in one case and a retardation in the other. For example, *a-c* and *a-d*, while each have an angular displacement of 30 degrees, will not parallel with each other because the secondary voltages are out of phase by 60 degrees. From an inspection of this table it is evident that the following combinations will parallel, as far as polarity is concerned.

a—*a* with *b*—*b*, *c*—*c*, and *d*—*d*	180°	angular displacement
a—*b* with *b*—*a*, *c*—*d*, and *d*—*c*	0°	angular displacement
a—*c* with *b*—*d*, *c*—*b*, and *d*—*a*	30°	angular displacement (advance)
a—*d* with *b*—*c*, *c*—*a*, and *d*—*b*	30°	angular displacement (retardation)

When it has been determined that two three-phase transformers have the same angular displacement of the secondary voltage compared to that impressed on the primary side and therefore can be operated in parallel, it remains to be determined which leads should be connected together on both the primary and secondary sides. In order to expedite this connection of leads it is customary to mark them so as to indicate the phase sequence of their voltages. If, for example, the leads issuing from the high-voltage side of a transformer are marked *A*, *B*, *C*, and the low-voltage leads are marked *A′*, *B′*, *C′*, the letters can be so placed on the

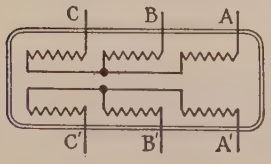


FIG. 112. Marking of leads on a star-star connected three-phase transformer.

leads as to indicate the phase sequence of the voltages as shown in Fig. 112. In connecting the two transformers in parallel, like-lettered leads are connected together on both the high-voltage and low-voltage sides. The phase sequence of the leads may be traced out in several ways but the most

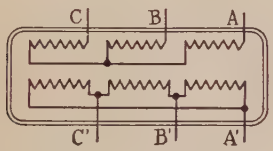


FIG. 113. Markings of leads on a star-delta connected three-phase transformer.

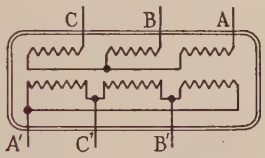


FIG. 114. Bringing the lead A' of a star-delta connection out at a different position relative to leads B', C'.

obvious one is by the use of Fig. 111, which indicates that similarly located leads should have the same letter on all transformers. For example, if the high-voltage leads are marked A, B, C, as shown in Fig. 113, the low-voltage leads should be marked as indicated. Had the lead marked A' on the low-voltage side been brought out the other side of the B', C' leads, it should be so marked as shown in Fig. 114.

CHAPTER XXI

PARALLEL OPERATION

THE PARALLEL operation of two or more transformers implies their being connected to carry the load jointly, so that an individual unit may be added to or taken away from the group with no other effect than decreasing or increasing the load on the remaining transformers. The measure of the satisfactory parallel operation of a number of transformers is the manner in which they divide the common load. If the individual units have the same rating they should share the load equally, and if they are different in rating, the load taken by each should be proportional to its rating.

In order that two transformers may operate satisfactorily in parallel, their secondary voltages must be in time-phase relation and equal in value. These conditions are established at no-load, if the polarity and voltage ratios of the transformers are the same. Another way of looking at this matter is to think of the secondary windings connected in parallel as a series circuit. Satisfactory parallel operation will be established, if there is no tendency to establish a current in this series circuit; which will be when the voltage of one winding is equal and in time-phase opposition to that of the other. If the voltages were not in time-phase opposition, a large current would flow between the transformers which would be in effect a short-circuit on both units. Under load conditions, the division of the load depends upon the relative regulation characteristics of the transformers.

DIVISION OF LOAD IN TRANSFORMERS CONNECTED IN PARALLEL

When connected in parallel, two transformers having the same polarity and ratio of transformation will divide the common load between them, in such proportion that the impedance drop across each is the same. If the impedance

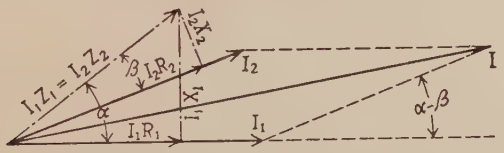


FIG. 115. Phase relations of the load currents and the voltage drops in two transformers connected in parallel.

drop across the first transformer is I_1Z_1 and across the second I_2Z_2, this relation may be expressed as follows:

$$I_1Z_1 = I_2Z_2, \tag{174}$$

and is shown graphically in Fig. 115. The impedance of transformers is usually specified in percent impedance volts, rather than in ohms, so that the following relations are developed with this fact in view. By definition, the percent impedance of the first transformer is

$$\text{Percent } Z_1 = \frac{(I_1 \text{ normal})\ Z_1}{E}, \tag{175}$$

where (I_1 *normal*) and E are the normal current and voltage of the transformer. Similarly for the second transformer,

$$\text{Percent } Z_2 = \frac{(I_2 \text{ normal})\ Z_2}{E}. \tag{176}$$

Substituting the values of Z_1 and Z_2 from equation 175 and 176 in equation 174 gives

$$\frac{I_1}{I_2} = \frac{(I_1 \text{ normal}) \text{ percent } Z_2}{(I_2 \text{ normal}) \text{ percent } Z_1}.$$

Multiplying through by E gives

$$\frac{(\text{k.v.a.})_1}{(\text{k.v.a.})_2} = \frac{(\text{K.V.A.})_1 \,\%\, Z_2}{(\text{K.V.A.})_2 \,\%\, Z_1}, \tag{177}$$

in which *k.v.a.* represents the load and *K.V.A.* the transformer rating.

In Fig. 115 the current I_1 and the voltage drop I_1R_1 are in phase, as also are the current I_2 and the voltage drop I_2R_2. The resistances R_1 and R_2 are the resistances of the first and second transformers respectively. The total current I is the vector sum of the currents I_1 and I_2, that is,

$$I = I_1 + I_2,$$

or

$$I = I_1 + I_2 \cos(\alpha - \beta) + jI_2 \sin(\alpha - \beta).$$

Multiplying through by E gives

$$k.v.a. = (k.v.a.)_1 + (k.v.a.)_2 \cos(\alpha - \beta) + j(k.v.a.)_2 \sin(\alpha - \beta).$$

Substituting value of $k.v.a._2$ from equation 177 gives

$$(\text{k.v.a.})_1 = \frac{\text{k.v.a.}}{1 + \frac{(\text{K.V.A.})_2 \,\%\, Z_1}{(\text{K.V.A.})_1 \,\%\, Z_2} \cos(\alpha - \beta) + j \frac{(\text{K.V.A.})_2 \,\%\, Z_1}{(\text{K.V.A.})_1 \,\%\, Z_2} \sin(\alpha - \beta)}. \tag{178}$$

Similarly,

$$(\text{k.v.a.})_2 = \frac{\text{k.v.a.}}{1 + \frac{(\text{K.V.A.})_1 \,\%\, Z_2}{(\text{K.V.A.})_2 \,\%\, Z_1} \cos(\alpha - \beta) + j \frac{(\text{K.V.A.})_1 \,\%\, Z_2}{(\text{K.V.A.})_2 \,\%\, Z_1} \sin(\alpha - \beta)}. \tag{179}$$

In using these equations in a particular case it is con-

venient to use the relations

$$\text{Cos } (\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

or

$$\text{Cos } (\alpha - \beta) = \frac{R_1}{Z_1}\frac{R_2}{Z_2} + \frac{X_1}{Z_1}\frac{X_2}{Z_2},$$

which has the same value as

$$\frac{(\%\text{ copper loss})_1\,(\%\text{ copper loss})_2 + (\%\text{ reactance})_1\,(\%\text{ reactance})_2}{\%\,Z_1\,\%\,Z_2}. \quad (180)$$

It is evident from equation 180 that if the first transformer has zero resistance, and the second transformer has zero reactance, that the cosine of the phase angle between the currents is equal to zero; or that the two currents are 90 degrees apart in phase. In this case the line current is 71 percent of the numerical sum of the currents in the trans-

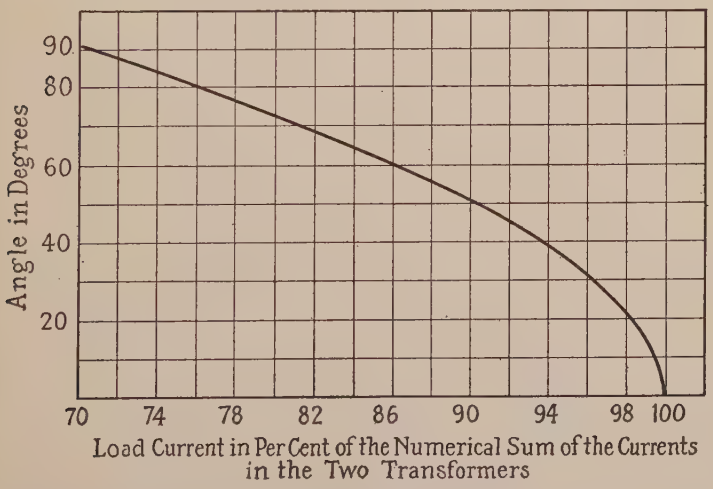


FIG. 116. Curve showing how the vector sum of the currents in the two transformers connected in parallel compares to the numerical sum with variable phase angle between the currents.

formers. This of course is a limiting case, and will not actually occur. It is also apparent from an inspection of Fig. 115 that if the *percentage reactance/percentage resistance* is the same for both transformers, the line current will be the numerical sum of the two transformer currents. This is also a limiting case, being the opposite limit to the previous one, but is a possible condition. A particular case of parallel operation will come somewhere between these two limits. Fig. 116 shows a curve drawn between the line current expressed as a percentage of the numerical sum of the two transformer currents and the phase angle between the transformer currents. From this curve it is evident that for differences in phase angle between the transformer currents of 15 degrees or less, the line current may be assumed to be the numerical sum of the transformer currents without serious error.

Example 64: How will the common 75 k.v.a. load be divided between a 25 k.v.a. transformer which has a copper loss of 1.5 percent and an impedance of 2.5 percent, and a 50 k.v.a. transformer which has a copper loss of 1.1 percent and an impedance of three percent?

The reactance of the first transformer will be two percent and of the second 2.8. The angle $(\alpha - \beta)$ is, from equation (180),

$$\cos(\alpha - \beta) = \frac{1.5 \times 1.1 + 2 \times 2.8}{2.5 \times 3} = 0.965.$$

Then from equation (178),

$$(\text{k.v.a.})_1 = \frac{75}{1 + \frac{50}{25} \times \frac{2.5}{3} \times 0.965 + j\frac{50}{25} \times \frac{2.5}{3} \times 0.259} = 28.2,$$

$$(\text{k.v.a.})_2 = 75 - 28.2 = 46.8.$$

In this example no appreciable error would have been introduced by the assumption that the line current was the numerical sum of the transformer currents, the angle $\alpha - \beta$ being approximately 15 degrees.

Example 65: To take a more extreme case, suppose it is desired to parallel a two-K.V.A. transformer having a copper loss of 2.25 percent, reactance of 1.1 percent and impedance of 2.5 percent, with a 50-K.V.A. transformer

having a copper loss of one percent, impedance of 3.5 percent, and reactance of 3.35 percent.
From equation 180,

$$\text{Cos } (\alpha - \beta) = \frac{2.25 \times 1 + 1.1 \times 3.35}{2.5 \times 3.5} = 0.68.$$

In this case the phase relation of the two currents cannot be neglected, as the angle $\propto - \beta$ is approximately 47 degrees.

USE OF BALANCE COILS IN PARALLELING TRANSFORMERS

It has been shown that the load current in two transformers connected in parallel so divides that the terminal voltages of the two units are equal in value and opposite in time-phase. In case this distribution of the current is not that desired, it is possible, by inserting a balance coil at *B*, in Fig. 117, to force the transformers to divide the load between them in any ratio required.

The balance coil is a single continuous winding on an iron core, having a tap at some point between its two ends. This tap is so located that when the desired current distribution is obtained, the ampere-turns on the two sides of it are equal. Since the balance coil absorbs the voltage tending to produce a circulating current, this current is reduced to a value only sufficient to magnetize the core of the coil. The ampere-turns on the two sides of the tap neutralize each other so far as any magnetizing effect on the core is concerned. It is rather difficult to predetermine the value of the voltage tending to circulate current for a particular case of parallel operation. For this reason the balance coil should be designed with a fairly low flux density in its core,

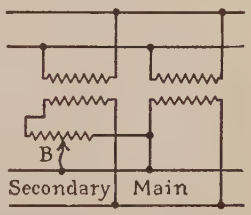


FIG. 117. Balance coil used with two dissimilar transformers to force the desired distribution of current between them.

so that if the voltage is greater than expected, the increased flux will not saturate the core sufficiently to require a large magnetizing current, which in this case will be the current circulating between the transformers. A balance coil may be used in paralleling three-phase transformers, the principles involved being the same.

CHAPTER XXII

OPERATING CONDITIONS

In service, transformers are subject to a number of operating conditions which are serious in that at times they interrupt the continuity of service. By observing the proper precautions, some of these troubles may be entirely eliminated and others may be minimized. A detailed study of some of these difficulties is important in suggesting improvements in manufacture, and to indicate when a change in the operating condition is desirable.

OVERLOAD

For distributing transformers the conditions of service are such that failures from overload are likely to occur. Since the maximum load exists for only a few hours daily, the tendency is to increase the maximum value of the load beyond safe limits, the transformers either being unprotected or being fused so that a heavy overload is permitted. A transformer burn-out often obliterates the evidence of the cause, and erroneous conclusions are difficult to avoid. However, in case of failure from overload, the windings are usually reduced to a mass of copper wire and charred insulation.

About eighty percent of the distributing transformers which burn out are below 7.5 k.v.a. in capacity, and of these about sixty-five percent fail from overload, twenty percent from water penetrating the windings and fifteen percent from short-circuits due to lightning and other causes.

SHORT-CIRCUITS AND GROUNDS

Short-circuits and grounds are usually due to lightning surges, to the presence of water in the transformer, to the wires working against each other when the windings expand and contract under varying conditions of load, or to the mechanical shocks to which the windings are subjected by short-circuits on the line. A short-circuit in service, say in the high-voltage winding, causes a large current to flow in the short-circuited part, which acts as a secondary carrying a large current, and draws an abnormally large current through the remainder of the primary winding. Thus the short-circuit starting in the primary winding may or may not spread through the entire coil, depending on the way the transformer is protected with fuses. The primary winding may be practically destroyed and the secondary coils remain in fairly good condition, and for this reason it usually is possible to distinguish between a short-circuit starting in the primary winding and a general roasting by overload. If the short-circuit starts in the secondary winding, the trouble is more likely to be localized in that part, as the immediate results of the short-circuit are more violent; and the primary coils may not be injuriously heated, depending on the completeness of the fuse protection. With a transformer of fairly large size the short-circuit is usually localized, and the transformer is cut out by the protecting device before the trouble has extended throughout the windings.

Before they are installed, transformers are sometimes stored in damp places, where they collect sufficient moisture inside of the cases to become visible, and bright metal parts like the heads of screws may become rusty. Water also gets into transformers by the so-called breathing action, which continually takes place with changes in load, particularly in damp localities, or may be put in with the oil. The higher the voltage of the transformer the greater is the

probability that a given amount of moisture will cause trouble. However, it is surprising how much water a comparatively low-voltage transformer may accumulate in service, and still continue to operate. It is not uncommon to find a considerable amount of water in the bottom of a transformer when it is operating satisfactorily. The heat generated within the coil does not permit the water to penetrate to a vital point, but if the transformer is allowed to cool and then put on the line again, the water may have penetrated sufficiently to cause an immediate failure. Therefore, if a transformer is put into service and a failure does not occur at once, a later breakdown due to moisture is not to be expected unless the conditions become very much worse. Where a short-circuit is started by moisture, the most superficial dielectric strength tests on some of the insulating material will confirm its presence. When a transformer is suspected of having absorbed moisture, although it may not be actually visible, a comparison of its insulation resistance with that of a dry transformer will indicate its actual condition. When moisture is known to be present, the only safe procedure, particularly if the transformer has a relatively high voltage, is to dry it out before it is installed.

DRYING OUT TRANSFORMERS

Relatively large masses of insulating material are used between high- and low-voltage windings and between windings and iron. When moisture is present throughout the transformer, the drying out process is slow because the moisture must be drawn out through this material. Transformers are usually dried by short-circuiting the low-voltage terminals and impressing such a voltage on the high-voltage leads as will circulate sufficient current to heat the coils gradually to a temperature by thermometer of about eighty-five degrees C. Ordinarily currents less than normal full load,

depending on the size of the transformer, are required; for example, with the larger sizes, say above ten k.v.a., the current required will be relatively smaller than for a one k.v.a. unit, the temperature by thermometer being the guide to the permissible current. Usually the applied voltage will be less than three percent of the normal voltage of the winding to which it is applied, as the average impedance is in the neighborhood of three per cent.

When the insulation resistance measurements indicate that the insulation from the high-voltage windings to ground is dry, it is fairly certain that the remaining insulation in the coils is also dry. A single insulation resistance measurement will not indicate definitely whether a transformer is sufficiently dry. A fairly reliable indication is given by a series of resistance readings taken during the drying run, plotting resistance values as ordinates and time as abscissae. Even then there is no definite knowledge of the point on the curve where danger ends and safety begins. There is a rather wide range between the minimum safety value and the maximum value of the insulation resistance, but the drying should be continued until the curve tends to become flat at a considerable elevation above its lowest part. This will require several hours on small units and a longer time with larger units. Where there is much moisture present, the curve will remain flat for a considerable period before starting to rise.

After having been soaked in oil, a transformer will have a lower insulation resistance than the same transformer without the oil. When oil is present, the rising of the curve when drying is due to getting rid of the oil in the form of vapor as well as to the removal of the moisture. When the curve is carried to a high point, both the oil and the moisture must be driven out, which takes a much longer time than if the oil was not present.

CARE OF INSULATING OIL

Oil in large quantities is generally shipped in tank cars, which are usually lagged to prevent rapid fluctuations in temperature during transit. This reduces the effect of expansion and contraction of the oil, with its resultant absorption of moisture. Oil is also shipped in steel barrels or drums with screw bungs, which are sealed before shipment. Such barrels should be packed on their sides with the screw bungs down. Small quantities of oil are shipped in soldered cans which are sealed before shipment. Every precaution should be taken in the shipment of oil to prevent absorption of moisture.

In spite of the precautions taken to make the containers tight, oil in drums or cans should not be stored in the weather, and oil drums should not be stored on end. Such a position allows water to collect in the head around the bung, and under such conditions even the tightest tank will allow water to enter. If possible, oil should be stored in a closed room, but if it is necessary to store it out doors, protection from the weather should be provided. Cans or barrels should not be unsealed before the oil is needed, as any change in temperature will cause an exchange of air in the receptacle with danger of precipitation of moisture into the oil.

During the transfer of the oil from the drums to the apparatus, care should be taken to prevent any moisture or dirt getting into either. A drum of cold oil when taken into a warm room will "sweat" and the resulting moisture on the outer surface may mix with the oil as it flows from the drum. Before breaking the seal the drum should therefore be allowed to stand long enough to reach room temperature. All vessels used for transferring the oil should be inspected to see that they are dry and free from dirt. Foreign material, such as scale from iron barrels, may become dislodged during shipment; or if the apparatus is installed,

for example, near cement mills, dust may enter the oil and reduce its dielectric strength.

Oil which has not been put through a filter press should be strained through two or more thicknesses of muslin or other closely woven cotton cloth, which has been washed and dried to remove the sizing. The straining cloths should be renewed as often as necessary, and not less than one set of cloths should be used for one transformer. Metal hose is preferable to rubber hose because the oil may dissolve the sulphur contained in the rubber. Oil should preferably be transferred from its container to the apparatus in dry weather. If it is necessary to do this in wet weather, the transfer should be made indoors to prevent the possible access of water. Before the apparatus is filled the oil should be sampled and tested. After the apparatus is filled with oil, it should immediately be covered and then allowed to stand sufficiently long to allow the oil to penetrate the apparatus. It should then be inspected and refilled to the proper oil level, if this has been appreciably lowered, and the cover be immediately tightened in position.

MECHANICAL STRESSES ON SHORT CIRCUIT

Due to the low reactance of transformers, a short-circuit on the secondary side would theoretically permit from twenty to fifty times full-load current to circulate through the windings, depending on their impedance, provided the supply system has sufficient capacity. Since the mechanical forces[1] on the windings, set up electromagnetically, vary as the square of the current, the forces may be sufficient under certain conditions to distort the windings and destroy the insulation not only between the conductors but also between the high- and low-voltage windings. To avoid failures of this kind the transformers should be thoroughly protected

[1] See also an article on "Mechanical Stresses in Transformers" by J. F. Peters in the *Electric Journal* for Dec. 1915, p. 555.

by fuses or circuit breakers, the windings be so thoroughly braced as to withstand the shocks of short-circuit or the impedance be made high enough to limit the current on short-circuit to safe values. Under actual operating conditions, transformers are seldom installed where the regulation of the source of supply, assuming that no circuit breaker or fuse protection is used, is such that the extreme current values just referred to actually occur. This is particularly true regarding distributing transformers, where the regulation of the distributing lines will greatly reduce the voltage impressed on the transformer under short-circuit conditions.

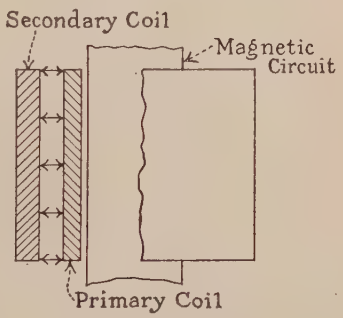


FIG. 118. Mechanical stresses between primary and secondary in a concentric transformer winding.

The direction of the forces between the primary and secondary winding of a concentric transformer winding is shown in Fig. 118. The mechanical force between these two coils is due to the leakage field between them, which has been discussed in chapter IX. From equation (87) the flux density of the leakage flux in the gap between the coils in gausses is

$$B = \frac{4\pi}{10}\frac{IT}{l},$$

where IT is the ampere turns in the winding and l is the average length of the leakage path in centimeters. Expressing the value of B as a maximum gives

$$B = \sqrt{2}\frac{4\pi}{10}\frac{IT}{l}. \qquad (181)$$

Since the magnetic flux density in the outer coil for ex-

ample is a maximum on the edge adjacent to the gap between the coils and is zero at the outer edge, the average value of the density in the coil is

$$B = \sqrt{2}\frac{2\pi}{10}\frac{IT}{l}. \tag{182}$$

The force in dynes acting on a group of conductors carrying current when in a magnetic field is

$$F = \frac{B\sqrt{2}ITl_m}{10}, \tag{183}$$

where I and T have the same values as before and l_m is the length of the mean turn of the conductor when wound into a transformer coil. Combining equations (182) and (183) gives

$$F = \frac{4\pi}{100}\frac{(IT)^2 l_m}{l}. \tag{184}$$

Assuming that I has a sine wave form, then F is a sine^2 function and the average force over one-half cycle is one-half of equation (184), or

$$F_{ave} = \frac{2\pi}{100}\frac{(IT)^2 l_m}{l},$$

where l and l_m may be expressed either in inches or centimeters. Expressing the force in pounds,

$$F_{ave} = \frac{1.41}{10^7}\frac{(IT)^2 l_m}{l}. \tag{185}$$

The force on the inner coil is towards the center, and is therefore one of compression. The force on the outer coil is away from the center and is therefore one of tension. If the electrical centers of the primary and secondary coils coincide, there is no force tending to move the coils in a

direction parallel to their axis, and therefore no necessity for bracing against end thrust. If the coils are rectangular, the force between coils tends to force the outer coil into a cylindrical shape.

Example 66: What is the force in pounds between the primary and secondary coils of a 100 k.v.a., 2,300 to $^{230}/_{115}$ volt, 60-cycle core type transformer, which has the following constants? The coils on each leg of the magnetic circuit are arranged in a single high-low group.

$I = 436$ amp., $l_m = 33$ in.,
$T = 12$, $l = 10$ in.

From equation (185),

$$F_{ave} = \frac{1.41}{10^7} \frac{(436 \times 12)^2 \times 33}{10} = 12.7 \text{ pounds.}$$

Assuming that the transformer was not protected by circuit breakers or fuses, that its impedance was five percent, and that the source of supply had a perfect regulation, the current on short-circuit would be 100/5 or 20 times normal. The force between the coils would be $20^2 \times 12.7$ or approximately 5,100 pounds. Actually, due to the regulation of the source of supply and of the distributing lines, the current would be something like 10 times normal, and the force between the coils $10^2 \times 12.7$ or 1,270 pounds.

When the electrical centers do not coincide, the mechanical force tending to separate the primary and secondary coils can be resolved into two components, one radial and the other parallel to the axis of the coils. It requires only a slight shifting of these centers to produce a large force tending to move the coils in a direction parallel to their axis. With the electrical centers shifted as shown in Fig. 119, the total force will be as shown by equation (185), but will be distributed as follows:

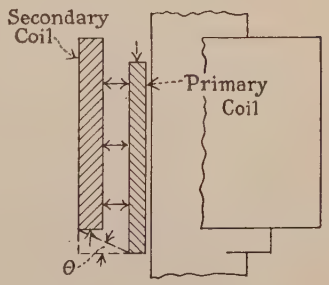


FIG. 119. Mechanical stresses when the primary and secondary windings are displaced axially.

$$\left.\begin{aligned} &\textit{The radial force} = F_{ave} \cos\theta, \\ &\textit{The end thrust} = F_{ave} \sin\theta. \end{aligned}\right\} \qquad (186)$$

Example 67: Suppose that in the preceding example the electrical centers of the coils had become displaced in an axial direction a distance of 0.5 inch, and that the radial distance between the centers was 2 inches, then

$$\cos\theta = 0.97,$$
$$\sin\theta = 0.24.$$

The forces under normal conditions are:
In the radial direction, $12.7 \times 0.97 = 12.3$ pounds.
In the axial direction, $12.7 \times 0.24 = 3.2$ pounds.
The forces under short circuit are:
In the radial direction, $1{,}270 \times 0.97 = 1{,}230$ pounds.
In the axial direction, $1{,}270 \times 0.24 = 320$ pounds.

CHAPTER XXIII

YEARLY COST OF OPERATION

In the following analysis certain assumptions have been made regarding the cost of power, cost of transformers and conditions of load, which may not conform exactly to the conditions for any particular case. Such costs are subject to considerable variation from time to time and the values used in the examples given are the ones prevailing when this was written.

COST OF ELECTRICAL ENERGY

The cost of making electrical energy is usually thought of as being made up of two main elements:

1. Production cost.
2. Fixed charge.

The items making up the total production cost are those which vary with the amount of energy produced, such as fuel, lubricants, water, etc. The items making up the total fixed charge are those which are practically independent of the total energy produced, such as the investment and administration charges. The investment charge is made up of interest, taxes, insurance and depreciation on the power station and distributing system of sufficient capacity to supply the maximum demand of the total connected load.

The investment charge equals the rate of annual charge times the total cost of the equipment. The rate of annual charge is usually made up as follows:

5.0 percent for interest.
1.5 percent for taxes.
1.5 percent for insurance.

3.0 percent for amortization.
Or a total of 11 percent.

The administration cost is for the salaries of officials and for office expenses in general.

The production cost is usually specified in cents per kilowatt-hour. It is more convenient to use this cost in dollars per watt year in the following calculations. To reduce cents per kw.-hr. to dollars per watt year, multiply by $(24 \times 365) \div 10^5$ or 0.0876.

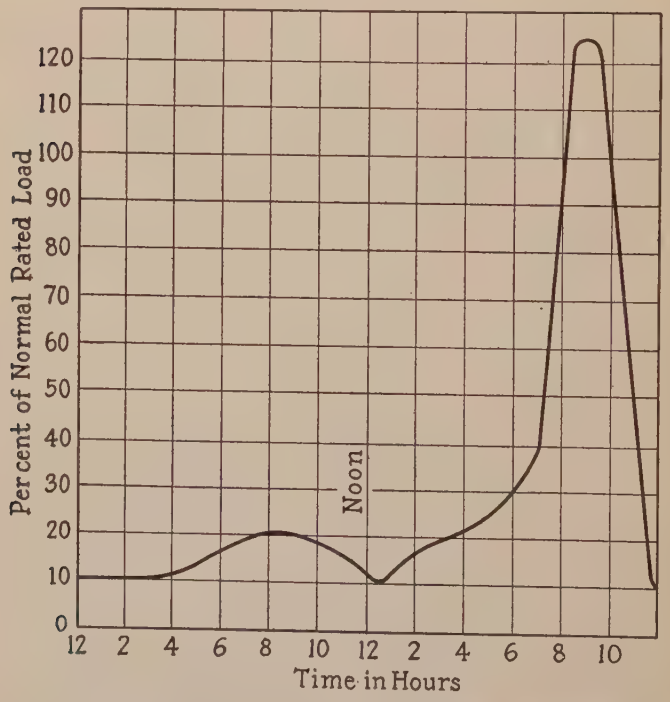


FIG. 120. Typical daily load curve on a five k.v.a. transformer. This load curve corresponds to the generally accepted understanding that the performance characteristics of distributing transformers should be based on a load condition of four hours at full load and twenty hours at no load. This gives a loss factor (mean square ordinate) of $4 \div 24 = 0.167$. Load factor of this curve is 0.24.

EFFECT OF LOAD FACTOR ON PRODUCTION COST AND FIXED CHARGE

A typical daily load curve for a distributing transformer is shown in Fig. 120, from which it is evident that the load factor is far from 100 percent. The production cost varies slightly, and the fixed charge to a considerable extent, with the load factor. A low load factor involves the use of a relatively large amount of equipment operating at light and inefficient loads. With a high load factor all the elements that enter into the production of power are operating at maximum economy, and the cost of power is a minimum. An exact definition of the 24-hour load factor for a period of one hour's maximum load is the ratio of the actual net output during that period to the product of the maximum hour output by the total numbers of hours in the period. Thus the daily load factor on this basis is

$$\text{Load factor} = \frac{\text{Actual net output for days of 24 hours}}{\text{Net maximum hour's load} \times 24},$$

or, expressed in another way, the load factor is the average daily use of the maximum demand, expressed as a percentage of 24 hours. The maximum demand is the maximum power taken, and may be based upon an instantaneous peak or cover a specified time.

Production cost has been found to vary approximately as the fourth root of the load factor.[1] Or the production cost at a given load factor is

$$p\left(\frac{1}{\text{Load factor}}\right)^{1/4},$$

where p is the production cost at unity load factor.

Example 68: If the production cost from Table VI for a 5,000 kw. plant is 0.347 cents per kw.-hr. for 100 percent load factor, what is the

[1] See "Standardization of Methods for Determining and Comparing Power Costs in Steam Plants," by Messrs. H. G. Stott and W. S. Gorsuch, *Proc. A. I. E. E.*, Vol. XXXII, p. 1,099.

production cost for a 24 percent load factor as shown by the curve in Fig. 120?

$$\text{Production cost} = 0.347\left(\frac{1}{0.24}\right)^{1/4} = 0.5 \text{ cents per kw.-hr.}$$

Since the load factor is the ratio of the average daily hours output to the maximum hours demand, a low load factor will require a large system to supply the maximum demand. For example, a system which delivers a certain amount of energy during a 24-hour period with a 25 percent load factor must be something less than four times the size of another system which supplies the same total energy in the same period with a 100 percent load factor. In the latter case the system will be less than four times the size in the former case, because with a low load factor the maximum demand exists only for a short time, for which the natural overload capacity of the system may be utilized. Assuming that the overload caacity of a station for short periods is

TABLE VI

COST OF POWER GENERATION

In cents per kilowatt-hour at 100 percent load factor[2]

Size of Station Kilowatts	Fuel	Labor	Oil Waste and Supplies	Repairs and Maintenance	Total Production Cost	Fixed Charges	Total
500	0.449	0.132	0.026	0.025	0.632	0.148	0.780
1000	0.364	0.094	0.019	0.021	0.498	0.124	0.622
2000	0.328	0.073	0.015	0.018	0.434	0.107	0.541
3000	0.304	0.065	0.013	0.016	0.398	0.095	0.493
4000	0.289	0.058	0.011	0.014	0.372	0.086	0.458
5000	0.271	0.053	0.010	0.013	0.347	0.080	0.428

25 percent, the normal capacity with a 25 percent load factor may be taken as $(1/0.25) \times (1/1.25)$ or 3.2 times the capacity of the system which operates with a 100 percent load factor. In the following examples it is assumed that

[2] From "Typical Electric Power Costs" by Mr. M. C. McNeil in the *Electric Journal* for March, 1914, p. 135.

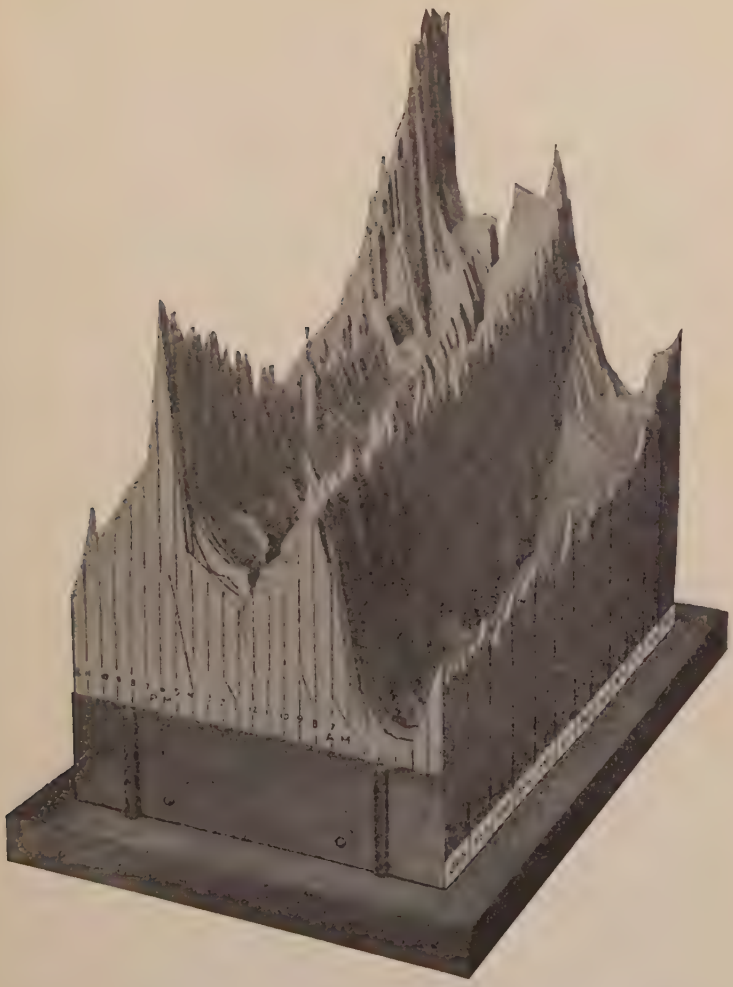


FIG. 121.* Grouping together the daily load curves, for a period of one year, of a central station supplying light and power. The seasonal variation of the points of maximum demand are shown by the changing heights of the peaks.

* Taken from *Proc. A. I. E. E.*, Vol. XXXVI, No. 12, page 942.

the fixed charge varies as the cost of the station which in turn depends on the load factor as indicated above.

Example 69: If the fixed charge is 0.08 cents per kw.-hr. for a 100 percent load factor, what is the proper fixed charge for a 24 percent load factor?

$$\text{Fixed charge} = 0.08 \times \frac{1}{0.24} \times \frac{1}{1.25} = 0.27 \text{ cents per kw.-hr.}$$

This assumes that the cost of the larger system is the same per kw. output as for the smaller one, and that the administration charge varies in the same way as the investment charge. Neither of these assumptions is strictly correct.

YEARLY COST OF TRANSFORMER LOSSES

The total yearly cost of the losses for a transformer, operated on the daily load curve shown in Fig. 120, will be

$$\text{Dollars per year} = p(L_i + FL_c) + d(L_i + L_c), \qquad (187)$$

where p is the production charge in dollars per watt year, d the total fixed charge including the investment and administration charges, expressed in dollars per year per watt of peak demand, L_i the iron loss of the transformer in watts, L_c is the copper loss at normal rating and F is the quantity which, multiplied by the copper loss at normal load, gives the actual continuous loss over the daily load curve. Therefore F is the mean square of the load curve when the ordinates of the load curve are expressed as percentages of normal load. The first term of equation (187) gives the total charge per year against the transformer on account of the actual energy consumed by the losses. The second term gives the annual charge on account of the extra equipment which must be provided to supply the energy during the peak load period. The cost of the transformer losses is figured in this way because the total cost of power for the entire station output must be the sum of the costs of the individual items making up the total load. For

convenience equation (187) may be written

$$\text{Dollars per year} = L_i(p + d) + L_c(pF + d). \qquad (188)$$

Example 70: What is the yearly cost of supplying the losses of a 5 k.v.a. transformer, on the daily load curve shown in Fig. 120, when $L_i = 42$ and $L_c = 100$.

This copper loss of 100 watts is based on a wattmeter measurement at 75 degrees C. in line with the A. I. E. E. rules. The average temperature of a distributing transformer, with a normal full-load temperature rise of 55 degrees C. operating on the load curve shown in Fig. 120, will doubtless be something less than 75 degrees C. and the assumption made is therefore subject to some slight criticism for this reason. With a load factor of 24 percent,

Production cost from Example 68 = 0.5 cents per kw.-hr.

Fixed charge from Example 69 = 0.27 cents per kw.-hr.

Since these figures are based upon costs at the central station, it will be assumed that the costs of electrical energy delivered to the distributing transformer is approximately 25 percent greater. Then,

Production cost = $0.5 \times 1.25 = 0.625$ cents per kw.-hr.

Fixed charge = $0.27 \times 1.25 = 0.338$ cents per kw.-hr.

When expressed in dollars per watt-year these figures become

Production cost = $0.625 \times 0.0876 = \$0.055$ per watt-year.

Fixed charge = $0.338 \times 0.0876 = \$0.0297$ per watt-year.

Therefore,

$$p = 0.055,$$
$$d = 0.030,$$
$$p + d = 0.085,$$
$$pF + d = 0.039.$$

Then from equation (188),

Dollars per year = $42 \times 0.085 + 100 \times 0.039 = 3.57 + 3.9 = \$7.47$.

Of the total cost of \$7.47 per year, \$3.57 is for the iron loss and \$3.90 is for the copper loss.

COST OF TRANSFORMER EXCITING CURRENT

The transformer exciting current causes a copper loss in the generating and primary distributing system, which is continuous as long as the transformer is connected to the line. Since it is continuous and a true energy loss, this copper loss must be placed in the same class as transformer iron loss. The loss varies with the magnitude and power-factor of the load; the greater the load and the lower the power-factor, the greater will be the copper loss.

The exciting current is composed of two parts, one which magnetizes the iron circuit and another which supplies the actual iron loss. Since the magnetic flux in the iron circuit of the transformer is a maximum when the impressed voltage is zero, and zero when the impressed voltage is a maximum, the impressed voltage and magnetic flux are 90 degrees apart in time phase relation. The magnetizing element of the exciting current, being in phase with the flux, is also at right angles to the impressed voltage and is therefore wattless. On the other hand, the eddy and hysteresis loss represents actual energy loss, which appears as heat in the magnetic circuit. The current which supplies this loss is in phase with the impressed voltage and is a working current. The total current on the primary side of the transformer from Fig. 122 is

$$I = [I_L^2 + m^2 I_L^2 - 2\, m\, I_L^2 \cos\{180^\circ - (\beta - \theta)\}]^{\frac{1}{2}},$$

where I_L is the primary load current due to the load current on the secondary side, and mI_L is the exciting current. In this case m is the ratio of the exciting current of the transformer to its normal full-load current. The total line loss is equal to the square of the current multiplied by the total resistance on the primary side of the transformer, including that of the generator, distributing lines and other intervening apparatus, such as step-up or step-down transformers. The total distributing loss is therefore

$$L_d = I_L^2 R + m^2 I_L^2 R + 2\, m\, I_L^2 R \cos(\beta - \theta),$$

where R is the total resistance on the primary side of the transformer as described above. If the transformer is operated only F percentage of the 24 hours at normal full-load, the total distributing loss becomes

$$I_L^2 RF + I_L^2 RFm\,[m + 2\cos(\beta - \theta)].$$

The part of this loss due to the normal primary working

current I_L is $I_L^2 RF$ and the part due to the exciting current is $mI_L^2 RF\,[m + 2\cos(\beta - \theta)]$. Expressing $I_L^2 R$ in terms of the power delivered in watts, it may be written

$$I_L^2 R = 1000cP,$$

where c is the ratio of the distributing copper loss to the total k.v.a. delivered and P is the k.v.a. rating of the transformer. The copper loss in watts due to the exciting current is then

$$L_d = 1000mcPF[m + 2\cos(\beta - \theta)].$$

The total output P may represent the output of any number of transformers or of a single unit. Expressing this in terms of the iron loss of the transformer as 100 percent gives

$$\text{Percent } L_d = \frac{1000mcPF[m + 2\cos(\beta - \theta)]}{L_i}. \qquad (189)$$

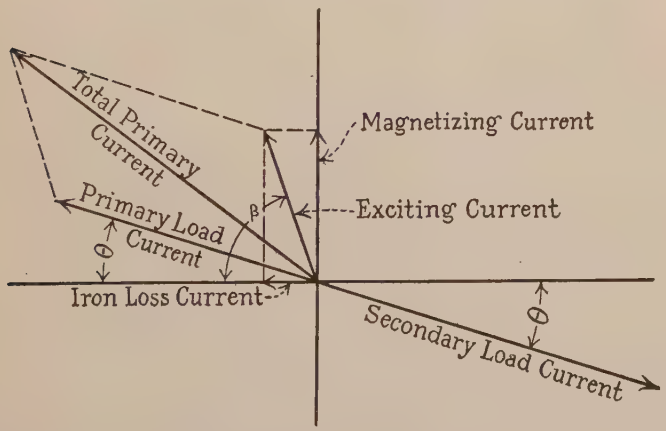


FIG. 122. Vector diagram, showing that the total current in the primary winding of a transformer is made up of the vector sum of the iron loss current, the magnetizing current and the load current.

The angle $(\beta - \theta)$ is determined as follows from Fig. 122, the primary voltage of the transformer being E,

$$\cos\beta = \frac{\frac{L_i}{E}}{m\overline{I_L}} = \frac{L_i}{1000mP}, \qquad (190)$$

and the angle θ is the one whose cosine is equal to the power-factor of the load. The data in Table VII will be found useful in numerical calculations.

Example 71: Let the following data refer to a group of transformers operating on a basis of four hours at full normal load and 20 hours at no-load.

For the exciting current ..... $m = 0.018$
For the distributing copper loss ..... $c = 0.12$
Iron loss ..... $L_i = 42$
Power-factor of load ..... $= 0.9$

Therefore since cos θ is 0.9, $\theta = 25$ degrees, 51 minutes. From equation (190),

$$\cos\beta = \frac{42}{1000 \times 0.018 \times 5} = 0.467.$$

Therefore $\beta = 62$ degrees, 11 minutes.
Then $\cos(\beta - \theta) = 0.805$.
From equation (189) the percent distributing loss now becomes

$$\frac{1000 \times 0.018 \times 0.12 \times 5 \times 0.167(0.018 + 2 \times 0.805)}{42} = 0.070.$$

That is, the loss in the system due to the transformer exciting current is equal to practically seven percent of the iron loss of the transformers, or a yearly cost of $\$3.57 \times 0.07 = \$0.25$.

TABLE VII

VALUES OF COS $(\beta - \theta)$

β (Degrees)	Cos β	Cos $(\beta - \theta)$				
		For 100% P-F	For 90% P-F	For 80% P-F	For 70% P-F	For 60% P-F
78	0.208	0.208	0.616	0.755	0.848	0.906
75	0.259	0.259	0.656	0.788	0.875	0.927
72	0.309	0.309	0.695	0.819	0.899	0.945
69	0.358	0.358	0.731	0.848	0.920	0.961
66	0.407	0.407	0.766	0.875	0.940	0.974
63	0.454	0.454	0.799	0.899	0.956	0.985
60	0.500	0.500	0.829	0.920	0.970	0.992
53	0.602	0.602	0.891	0.961	0.992	1.000

It is interesting to compare the cost of the exciting current as given in Example 71, which is four hours at full load and 20 hours at no-load, to the cost for continuous operation at full load. The latter cost is given in the following example, with all other conditions the same as for Example No. 71,

Example 72: From equation (189) the loss is

$$\frac{1000 \times 0.018 \times 5 \times 0.12 \times (0.018 + 2 \times 0.805)}{42} = 0.42.$$

This gives a yearly cost of the exciting current of $\$3.57 \times 0.42 = \$1.50$.

COST OF TRANSFORMER REGULATION

The current consumed by the load on a transformer is proportional to the voltage delivered. Therefore, since the power consumed by the load is proportional to the product of the voltage and the current, it is proportional to the square of the impressed voltage. If R be the regulation of the transformer, the percent of normal voltage delivered at full-load will be

$$\text{Percent of voltage} = \left[\frac{E - \frac{RE}{100}}{E}\right] = 1 - \frac{R}{100}.$$

The percentage of power delivered is therefore $[1 - (R/100)]^2$ and the actual power in watts is

$$\text{Actual power delivered} = \left(\frac{100 - R}{100}\right)^2 1000PF,$$

where P is the k.v.a. rating of the transformer, and F is the quantity which, multiplied by the normal rating of the transformer, will give the actual energy delivered during a given daily load curve. The case of 90 percent power-factor is assumed as being that of the average distributing circuit.

The energy not sold due to the regulation of the transformer is

$$= 1000PF - \left(\frac{100 - R}{100}\right)^2 1000PF$$

$$= \left[\frac{2R}{100} - \left(\frac{R}{100}\right)^2\right] 1000PF,$$

Multiplying by the profit per year realized in selling energy gives the yearly loss

$$\text{Dollars loss per year} = \left[\frac{2R}{100} - \left(\frac{R}{100}\right)^2\right] 1000PF\,k,$$

where k = Profit in dollars per watt year.

Since the value of $(R/100)^2$ is small compared to $2R/100$, this relation may be written:

$$\text{Dollars loss per year} = 20RPFk. \qquad (191)$$

Example 73: What is the yearly loss with a 5 k.v.a. transformer which has a regulation at 90 percent power-factor and full load of 2.67 percent, if the daily load curve is such that the loss factor is 0.167?

Assume that the profit realized from selling a watt-year of energy is approximately 50 percent of the cost of energy delivered to the transformer or say \$0.04 per watt-year, then from equation (191),

$$\text{Dollars loss per year} = 20 \times 2.67 \times 5 \times 0.167 \times 0.04 = \$1.76.$$

A part of this loss might be eliminated by the use of regulating devices tending to keep the voltage constant at the secondary terminals of the transformer. However, it is customary to take care of drop in primary voltage only by the use of a potential regulator.

TOTAL COST OF OPERATING A TRANSFORMER

The total cost of operating a transformer includes not only the cost of supplying its losses and a charge for its exciting current and regulation, but also the annual charge

on the money invested in the transformer. The total cost of operating the transformer therefore consists of the following items:

1. Investment charge.
2. Cost of iron loss.
3. Cost of copper loss.
4. Cost of exciting current.
5. Cost of regulation.

Example 74: What is the total cost of operating a 5 k.v.a. transformer whose cost is $51, with a rate of annual charge of 11 percent, all other conditions being the same as for the preceding examples?

1. Investment charge	$ 5.61
2. Cost of iron loss from Example 70	3.57
3. Cost of copper loss from Example 70	3.90
4. Cost of exciting current from Example 71	0.25
5. Cost of regulation from Example 73	1.76
Total	$15.09

Example 75: What is the total yearly cost of operating a 5 k.v.a. transformer, whose cost is $43.50 and whose performance characteristics are as follows:

$L_i = 58$, $L_c = 110$,
$m = 0.063$, Regulation at 90 percent power-factor $= 2.85$.

1. Investment charge	$ 4.78
2. Cost of iron loss	4.93
3. Cost of copper loss	4.30
4. Cost of exciting current	0.66
5. Cost of regulation	1.90
Total	$16.57

It is evident that, under the assumed conditions, the transformer in Example 74 is the most economical one to use. The difference in the yearly cost being $16.57 — $15.09 or $1.48, the cost of the transformer in Example 74 could be increased from $51.00 to $64.50, without its total yearly cost of operation exceeding that of the transformer in Example 75.

Example 76: Would it be economical to replace an old 5 k.v.a. transformer, which has the following performance characteristics, by the 5 k.v.a. transformer in Example 74,

$L_i = 75$, $L_c = 120$,
$m = 0.063$, Regulation at 90 percent power-factor $= 3.03$.

The additional investment is the cost of the new transformer, which is $51 plus say 10 percent of this value for making the change, minus about 20 percent for the scrap value of the old transformer, or a net amount of $45.90. Therefore,

The yearly cost of the new investment	$ 5.05
Total yearly cost of the losses and regulation of the old transformer	13.78
Total yearly cost of the losses and regulation of the new transformer	9.48

The yearly saving on the losses and regulation by the use of the new transformer would be $13.78 — $9.48 or $4.30. Since this is less than the cost of the additional investment of $5.05, it would not be advisable in this case for reasons of economy only, based on the assumed conditions, to scrap the old transformer and replace it with the new one.

CHAPTER XXIV

CONDITIONS FOR MINIMUM COST OF OPERATION

THE discussion in chapter XXIII related to the calculation of the total yearly cost of operating a transformer, while the present chapter is concerned with the relations of its various characteristics which will make the total yearly cost of operation a minimum. As in the previous case the conclusions reached depend upon the assumptions made as to cost of power, cost of transformer and conditions of load. Therefore, no general answer can be made which will be applicable to all cases, and if the data as to the average conditions of a large number of cases at a given time are obtained, the solution is still not a general one as the relative values of the various constants are continually changing.

RATIO OF COPPER TO IRON LOSS TO MAKE THE YEARLY COST OF THE LOSSES A MINIMUM

In order to get a definite idea as to the conditions which make the total cost of operation a minimum, with change of the ratio of the losses, the subject may be treated analytically as follows. The cost of the exciting current cannot easily be brought into the relation, but since its cost is the smallest element of the total, its omission does not greatly change the result.

If on the same frame the iron loss is increased, the exciting current increases. When the copper loss increases, the regulation becomes poorer, particularly with loads of high power-factor. The relation between the losses of a transformer of fixed output on a given frame, and therefore

of constant cost, as given in equation (47), is as follows:

$$L_c L_i = \text{Constant}, \tag{192}$$

where L_c is the copper loss by wattmeter at 75 degrees C., and L_i is the iron loss.

The relation between the losses is indicated by r when

$$r = \frac{L_c}{L_1}. \tag{193}$$

Combining equations (192) and (193),

$$L_1 = \frac{\text{Constant}}{\sqrt{r}}. \tag{194}$$

The costs of operating the transformer which vary with the ratio of the losses, aside from the exciting current are, from equations (188) and (191),

$$\text{Dollars per year} = L_i(p + d) + L_c(pF + d) + 20RPFk,$$

where p and d are the production and fixed charges respectively for producing electrical energy and F is the quantity which, multiplied by the copper loss at normal load, gives the actual continuous copper loss over the daily load curve.

Restricting the solution to a load of 100 percent power-factor, this may be written

$$\text{Dollars per year} = L_i(p + d) + L_c(pF + d) + 2L_cFk.$$

Substituting the value of L_c from equation (193) gives

$$\text{Dollars per year} = L_i[(p + d) + r(pF + d) + 2rFK].$$

Substituting the value of L_i from equation (194) gives

$$\text{Dollars per year} = \text{Constant}\left[\frac{(p+d)}{\sqrt{r}} + \sqrt{r}(pF+d) + 2\sqrt{r}Fk\right].$$

Differentiating this expression with respect to r and putting the differential coefficient equal to zero gives

$$r = \frac{(p + d)}{pF + d + 2Fk}. \qquad (195)$$

Example 77: What is the value of the ratio of the losses to make the cost of the losses a minimum, for a 5 k.v.a. transformer operating under the following conditions:

$p = 0.055$,
$d = 0.03$,
$p + d = 0.085$,
$pF + d = 0.039$,
Cost of transformer = \$51.00,
Annual interest charge, $i = 0.11$.
where $L_i = 42$; $F = 0.167$; $L_c = 100$; and $k = 0.04$.

From equation (195),

$$r = \frac{0.085}{0.039 + 0.0134} = 1.62.$$

The values of the losses to give this ratio are: $L_i = 51$ and $L_c = 82$.

To get the actual minimum cost of operating the transformer the cost of the exciting current must be taken into account. The cost of the exciting current for a transformer fully loaded continuously is a matter of some importance, but of smaller importance when the transformer is operated at full load only a portion of the time.

The total cost of operating the five k.v.a. transformer with different ratios of the losses, using the same frame in all cases, is given in Table VIII.

When the cost of the exciting current is taken into account, the ratio of the losses becomes closer to a value of 2 as shown by the table, than the case in Example 77, where the exciting current is ignored. Table VIII also gives the investment charge, which is a constant quantity and therefore does not affect the point of minimum total cost with variation of the ratio of the losses. In calculating this

table a five k.v.a. transformer frame was used which was designed for a ratio of the losses between 2 and 2.5. When this frame is worked at a magnetic density which raises its iron loss up to equality with the copper loss, a relatively high exciting current results. This keeps the point of minimum yearly total cost of operation at a higher ratio of copper to iron loss than would result if the frame were designed for a higher iron loss.

The striking point brought out by Table VIII is that the total cost of operating the transformer changes very slowly from a minimum charge of $15.04 for a ratio of 2, for

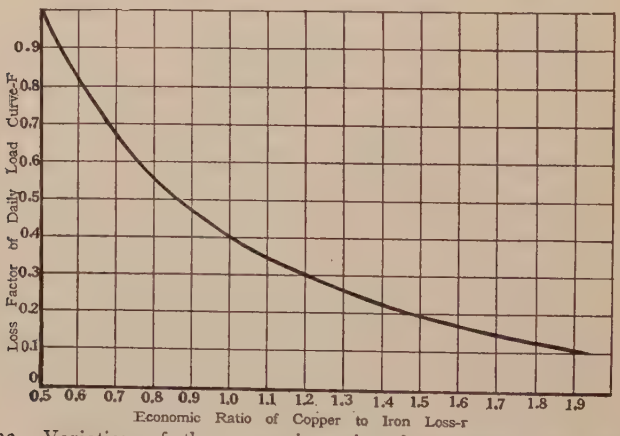


FIG. 123. Variation of the economic ratio of copper to iron loss with changes of loss factor of the load curve.

increasing or decreasing values of the ratio of the losses. For example, changing the ratio from 2 to 1.5, the cost increases $0.06 or less than one percent of the yearly cost of operation. Changing the ratio from 2 to 2.5, a correspondingly low increase in the yearly cost is found. This would indicate that attempts to secure low iron in distributing transformers has perhaps been overdone, when the low iron loss is secured by increasing the copper loss and regulation.

The really important item in comparing the relative economy of two transformers is the product of iron and copper losses, assuming of course that the exciting current has a reasonably low value, and the ratio of the losses is between 1.5 and 2.5.

It is evident from equation (195) that r increases when F or k decreases, and when p increases. Changes in the value of d will have a relatively small effect on the value of r. It is also evident that when F becomes unity, that is, the transformer is fully loaded continuously, the value of r becomes less than unity. Fig. 123 shows the variation of the value of r, with changes of F, when p, d and k have values as given in the examples already given.

THE MOST ECONOMICAL OUTPUT RATING OF A GIVEN TRANSFORMER

The examples which have been given consider transformers of fixed output and have calculated the yearly cost of operation including the investment charge, the cost of the losses and the regulation. It will be interesting to determine if a given transformer can be operated economically at other than its normal rated output. This can be analyzed by permitting the output of the transformer to vary and determining the rating which will make the total cost per k.v.a. output a minimum.

Example 78: What is the most economical output rating of the transformer covered by Example 77? This problem is analyzed in Table IX.

It is evident from Table IX that with the constants assumed, the transformer could be operated with a minimum cost per k.v.a. output somewhere between six and seven k.v.a. However, the curve is very flat in this region and the increased cost per k.v.a. output from seven to five k.v.a. is only about five percent. Also, aside from the question of economy, the regulation for the higher ratings becomes objectionable from the point of view of service, also questionable because of considerations of heating, and finally a normally rated 7.5 k.v.a. transformer will have a lower cost of operation per k.v.a. output than the five k.v.a. transformer covered by Example 78.

Example 79: What is the cost of operation of a 7.5 k.v.a. transformer

which has a cost of $66.00 and the following performance characteristics? $L_i = 57$; $L_c = 132$; $m = 0.0155$; and regulation at 90 percent power-factor = 2.63.

1. Investment charge	$ 7.26
2. Cost of iron loss	4.85
3. Cost of copper loss	5.15
4. Cost of exciting current	0.33
5. Cost of regulation	2.64
Total	$20.23

This gives a cost of $2.70 per k.v.a. output, which is less than the minimum cost per k.v.a. output of the five k.v.a. at an increased output in the neighborhood of seven k.v.a. Further, this 7.5 k.v.a. transformer does not have the objectionable features of over-heating or bad regulation as does the five k.v.a. unit when operated at the higher rating.

There are, however, cases arising in practice where there is only one transformer available for a particular service and in such cases it is important to know the most economical load which the unit should carry under the particular operating conditions. This load may be determined analytically as follows, but care should be used not to select a rating which will give an objectionable regulation from the point of view of service, or one which will overheat the transformer.

The total cost of operating the transformer is:

$$\text{Dollars per year} = iC + L_i(p + d) + K + L_c'(pF + d) + 2L_c'Fk,$$

$$\text{Dollars per year per k.v.a.} = \frac{iC + L_i(p + d) + K + L_c'[(pF + d) + 2Fk]}{P'}, \quad (196)$$

where P' is the most economical rating under the assumed conditions, C is the cost of the transformer in dollars, i is the investment rate, expressed decimally, K is the constant cost of the exciting current and L_c' is the copper loss, at operat-

ing temperature, associated with this rating. In substituting $L_c' \div P'$ for the regulation of the transformer, the solution has been restricted to a 100 percent power-factor load, as far as the effect of the regulation is concerned.

Evidently the value of P' desired is the one which will make the value of equation (196) a minimum. Before differentiating this equation with respect to P' as the variable, it will be necessary to obtain an expression for the variable L_c' in terms of P'. By definition $P' = E'I'$, where E' and I' are the voltage and current output associated with the output P'. Also $L_c' = I'^2R'$, where R' is the equivalent resistance of the primary and secondary windings of the transformer at operating temperature with load P'. From these two equations, $L_c' = (P'/E)^2R'$.

Substituting this value of L_c' in equation (196) gives

Dollars per year per k.v.a. =

$$\frac{iC + L_i(p + d) + K + \left(\frac{P'}{E}\right)^2 R'[(pF + d) + 2Fk]}{P'}.$$

Differentiating this expression with respect to P', and putting the differential coefficient equal to zero gives

$$P'^2 = \frac{E^2}{R'}\left[\frac{iC + L_i(p + d) + K}{(pF + d) + 2Fk}\right].$$

Multiplying through by the normal rated current I squared gives

$$P'^2 = \frac{E^2I^2}{I^2R'}\left[\frac{iC + L_i(p + d) + K}{(pF + d) + 2Fk}\right].$$

Since $P = EI$, this equation may be written

$$P'^2 = \frac{P^2}{I^2R'}\left[\frac{iC + Li(p + d) + K}{(pF + d) + 2Fk}\right].$$

Multiplying both numerator and denominator by R/R' gives

$$P'^2 = \frac{P^2 \dfrac{R}{R'}}{I^2 R}\left[\frac{iC + L_i(p+d)}{(pF+d)+2Fk}\right]. \qquad (197)$$

But $L_c = I^2R$, where L_c is the copper loss of the transformer in watts at normal output P and R is its equivalent resistance at the temperature resulting from operation at output P. Then equation (197) becomes

$$P' = P\left\{\frac{\dfrac{R}{R'}}{L_c}\left[\frac{iC + L_i(p+d)+K}{(pF+d)+2Fk}\right]\right\}^{1/2}. \qquad (198)$$

Example 80: What is the most economical output rating of the five k.v.a. transformer in Example 78?

From equation (198), assuming that the ratio R/R' is unity,

$$P' = 5\left\{\frac{1}{100}\left[\frac{0.11 \times 51 + 42 \times 0.085 + 0.25}{0.039 + 0.0134}\right]\right\}^{1/2} = 6.72.$$

It is obvious that the value of R/R', which is the ratio of the equivalent resistance of the transformer when operating at normal rating to the resistance when operating at an output of P', is less than unity. An exact determination of this ratio is practically impossible, but it will be assumed that the average temperature of the transformer in this particular case at an output of P' is 20 degrees C. higher than with an output of P. This gives a value for the ratio R/R' of approximately 0.93. Solving equation (198) again with this value of R/R' gives the most economical rating as 6.5 k.v.a.

This checks the result secured from Example 78. The value of the output rating given by Example 80 is somewhat greater than that given by 78, because equation (198) considers the regulation at 100 percent power-factor load while Example 78 contemplates regulation at 90 percent power-factor. The regulation at 90 percent power-factor being poorer than at 100 percent, the charge for regulation is

greater in the former case. An inspection of equation (198) indicates that the economical output rating increases with the investment, iron loss and exciting current costs, and decreases as the copper loss and regulation costs become greater.

RELATION OF THE COST OF A TRANSFORMER AND ITS LOSSES TO MAKE THE TOTAL COST OF ITS OPERATION A MINIMUM

The preceding discussion has made no reference to the relation existing between the cost of a transformer and its losses. In a particular case a given transformer might be economically replaced by one of higher cost and reduced losses, or one of lower cost with increased losses. In order to analyze this phase of the subject it is necessary to know the relation between the cost of the transformer and its losses. This relation may be expressed algebraically as follows, which is the same as that in equation (52),

$$C = \frac{K'}{(L_i L_c)^{3/2}} + C', \tag{199}$$

where K' is a constant and C' that part of the cost of a transformer which does not vary with its losses, as its design is changed.

The total yearly cost of operating a transformer aside from the cost of the exciting current has been shown to be

$$\text{Dollars per year} = iC + L_i(p+d) + L_c[(pF+d) + 2Fk]. \tag{200}$$

Substituting the value of C from equation (199) and the value of L_c from equation (2) in equation (200),

$$\frac{\text{Dollars per year}}{} = \frac{iK'}{L_i^3 r^{3/2}} + iC' + L_i\{(p+d) + r[(pF+d) + 2Fk]\}.$$

Differentiating this equation with respect to L_i and putting the differential coefficient equal to zero,

$$L_i = \left\{ \frac{3iK'}{r^{3/2}\{(p+d) + r[(rF+d) + 2Fk]\}} \right\}^{0.25}. \quad (201)$$

If we take r to be equal to 2, then

$$L_i = \left\{ \frac{1.06iK'}{(p+d) + 2[(pF+d) + 2Fk]} \right\}^{0.25}. \quad (202)$$

From equation (202) it is apparent that a considerable change in the quantity whose exponent is 0.25 is required to materially change the value of L_i. For example, increasing its value ten percent increases the value of L_i only two percent, and decreasing its value ten percent reduces the value of L_i only two percent.

Example 81: What should be the cost and losses of a five k.v.a. transformer, based on the condition given in the previous examples? The value of K' to be taken from a five k.v.a. transformer whose cost is $51.00, and whose losses are

$$L_i = 42 \text{ and } L_c = 100.$$

It will be assumed that the cost of the active material in the transformer is 65 percent of $51.00 or $33.25. From equation (199),

$$K' = 33.25(42 \times 100)^{3/2} = 905 \times 10^4.$$

Then from equation (202),

$$L_i = \left\{ \frac{1.06 \times 0.11 \times 905 \times 10^4}{0.085 + 0.105} \right\}^{0.25} = 49,$$

$$L_c = 2 \times 49 = 98.$$

The new cost from equation (199) will be

$$C = \frac{905 \times 10^4}{(49 \times 98)^{3/2}} + 17.75 = \$45.25.$$

An actual comparison of the cost of operating these two transformers gives the results shown in Table X.

TABLE VIII

Total Yearly Cost of Operating a Five K.V.A. Transformer

r	Iron Loss	Copper Loss	Percent Exciting Current	Percent Regulation	Investment Charge	Cost of Iron Loss	Cost of Copper Loss	Cost of Exciting Current	Cost of Regulation	Total Yearly Cost
1.0	64.8	64.8	6.0	2.04	$5.61	$5.51	$4.36	$0.22	$1.89	$15.27
1.5	53.0	79.2	3.0	2.29	5.61	4.51	4.00	0.24	1.79	15.12
2.0	45.8	91.7	2.2	2.52	5.61	3.90	3.57	0.29	1.67	15.04
2.5	41.0	102.5	1.75	2.71	5.61	3.48	3.09	0.38	1.51	15.10
3.0	37.5	112.0	1.6	2.87	5.61	3.19	2.53	0.66	1.35	15.66

TABLE IX

Total Yearly Cost per K.V.A. Output of Operating a Given Transformer

K.V.A. Output	Iron Loss	Copper Loss	Percent Exciting Current	Percent Regulation	Investment Charge	Cost of Iron Loss	Cost of Copper Loss	Cost of Exciting Current	Cost of Regulation	Total Cost	Cost per K.V.A.
4	42	64	2.25	2.14	$5.61	$3.57	$2.5	$0.25	$1.14	$13.07	$3.26
5	42	100	1.8	2.67	5.61	3.57	3.9	0.25	1.76	15.09	3.02
6	42	144	1.5	3.21	5.61	3.57	5.62	0.25	2.53	17.58	2.93
7	42	196	1.28	3.74	5.61	3.57	7.65	0.25	3.44	20.52	2.87
8	42	256	1.12	4.27	5.61	3.57	9.99	0.25	4.47	23.89	2.99

TABLE X

Comparative Costs of Operating Two Transformers

K.V.A.	Iron Loss	Copper Loss	Percent Exciting Current	Percent Regulation	Investment Charge	Cost of Iron Loss	Cost of Copper Loss	Cost of Exciting Current	Cost of Regulation	Total Yearly Cost
5	42	100	1.8	2.67	$5.61	$3.57	$3.90	$0.25	$1.76	$15.09
5	49	98	1.8	2.63	4.98	4.17	3.83	0.25	1.74	14.97

CHAPTER XXV

ECONOMICAL RATIO OF COPPER TO IRON LOSS

The problem of the determination, from the operating point of view, of the most economical ratio of the copper loss of a transformer to the iron loss is discussed at some length in chapter XXIV. The treatment given here is a more simple and general one, as it does not include all of the refinements in the previous chapter.

It is well known that the cost of making electrical energy is made up of two main elements:

1. Production cost.
2. Fixed charges.

The items making up the production cost are those which vary with the amount of energy produced, such as fuel, lubricants, water, etc. The items making up the total fixed charges are those which are practically independent of the total energy produced, such as the investment and administration charges. The investment charge is made up of interest, taxes, insurance and depreciation on a power station and distribution system of sufficient capacity to supply the maximum demand of the total connected load. The administration cost is for salaries of officials and for general office expenses.

In the following example it will be assumed that the production cost for a load having a 20 percent load factor is one cent per kilowatt hour and under the same conditions the fixed charge is one-half cent per kilowatt-hour. The term "load factor" used in this connection is the ratio of the average daily output to the maximum hour's demand. The production cost is usually specified in cents per kilowatt-

hour, but it is more convenient to use this cost in dollars per watt-year in the following calculations. To reduce cents per kilowatt-hour to dollars per watt-year, multiply by $(24 \times 365) \div 10^5$ or 0.0876. Multiplying the above production and fixed charges by 0.0876 gives

Production charge .....$0.0876 per watt-year
Fixed charge .....$0.0438 per watt-year

Example 82: What is the yearly cost of operating a 50 k.v.a., 60 cycle, 2,300 to 230-115 volt transformer, which has an iron loss of 260 watts and a copper loss by wattmeter at 75 degrees C. at normal rated output of 600 watts?

Since the iron loss is continuous and the copper loss exists for only four hours per day or 4/24 of the time, the yearly production cost of the iron loss in the transformer is 0.0876×260 or $22.78, and the cost of the copper loss is $0.0876 \times 4/24 \times 600$ or $8.76, and the total production cost is $31.54.

Since both the iron loss and the copper loss occur at the peak load period, the yearly fixed charge for the losses, or the annual charge on account of the extra equipment which must be provided to supply this energy during the peak load period is $0.0438 \times (260 + 600)$ or $37.67. The total yearly cost of the losses is therefore $31.54 plus $37.67 or $69.21. The cost of the transformer losses is figured in this way because the total cost of power for the entire station output must be the sum of the costs of the individual items making up the total load.

Table XI gives the production cost, fixed charge and total cost for the losses resulting from the variation of the ratio of the copper to the iron loss. The exact values of the losses resulting from the variation of these ratios are based on the fact that the product of the losses remains constant if the different designs of the transformer to secure these losses are made with the same frame. With this assumption, the cost of the transformer is a constant quantity. Table XI indicates that, with increasing values of the ratio of the losses, the production charge slowly decreases until a ratio of 6 is secured, above which it again gradually increases. The fixed charges have a constantly increasing value as the ratio increases. The sum of the production cost and fixed charge therefore becomes a minimum somewhere in the

region of 2.3, as shown by Table XI. The values in the table are plotted in Fig. 124. The lower curve is for the produc-

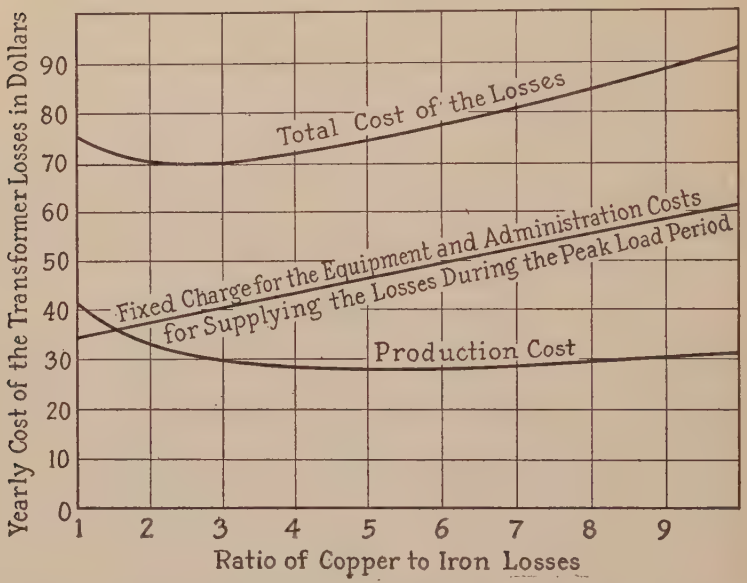


FIG. 124. Curves showing the most economical ratio of the copper to the iron loss for a 50 k.v.a. distribution transformer.

tion cost, and while it reaches a minimum value at a ratio of the copper to the iron loss of 6, the curve is very flat and the production cost changes only through a range of about five percent of its value, for a variation of the ratio of the losses from 3 to 8. It is not correct, therefore, even on the basis of the production cost alone, to say that one watt of iron loss is equal to six watts of copper loss.

The upper curve shows the total cost of supplying the transformer losses, and each point is the sum of the corresponding points on the two curves representing the production cost and fixed charge. The curves show how the

addition of the constantly increasing value of the fixed charge to the production cost curve reduces the point of minimum cost from a ratio of the losses of 6 to approximately 2.3. It is also apparent that the total cost curve is fairly flat and the cost does not materially change for a range of variation of the loss ratio from 2 to 3.

It might be argued that general conclusions should not be drawn from the particular values used for the production cost and fixed charge for power, in the example given. A little consideration, however, will show that the figures used for these quantities do not fix the shape of the production

TABLE XI

ANNUAL COST OF TRANSFORMER LOSSES OF A 50 K.V.A. TRANSFORMER

Ratio of Copper to Iron Loss	Iron Loss	Copper Loss	Production Cost of Transformer Losses	Fixed Charge for Transformer Losses	Total Cost of Transformer Losses
1	395	395	$40.30	$34.70	$75.00
2.3	260	600	31.54	37.67	69.21
6	162	965	28.20	49.50	77.70
10	125	1250	32.50	60.30	92.80

cost curve or the slope of the fixed charge line, but merely their position relative to each other. If the fixed charge is small compared to the production cost, the total cost curve will approximate that of the production cost curve, giving the minimum total cost for a relatively large ratio of the losses. On the other hand, if the fixed charge is large compared to the production cost, the total cost curve will approximate that of the fixed charge curve, giving the minimum total cost at a relatively small value of the ratio of the losses.

Two factors which have a bearing on the proper relation of the losses have not been taken into account. These are the copper loss on the primary side of the transformer due to

its exciting current and the regulation of the transformer itself. The exciting current increases with the transformer iron loss but, while the primary copper loss due to this current is in the same class as iron loss, the amount by which the iron loss is thus increased would ordinarily have a small effect in increasing the most economical ratio of the losses.

The regulation of the transformer becomes poorer as the ratio of the losses increases. As a poor regulation means a loss of revenue due to a reduced amount of energy delivered on account of the fall in voltage, it is apparent that the cost of the regulation is a factor tending to reduce the ratio of the transformer losses. It is sometimes argued that induction regulators are ordinarily used to keep the secondary voltage of the transformers at a definite value. However, it is practically impossible to keep the secondary voltage on a distributing system of any size at a constant value. The U. S. Bureau of Standards has made the following statement bearing on this point.[1]

"On alternating-current circuits, good regulation is most readily accomplished by voltage, or potential regulators, either of the induction or step-by-step type. Induction regulators can ordinarily be relied upon to maintain a uniform voltage at all loads at the center of distribution to within one percent. As the peak load will usually occur at different times on different feeders, and as the necessary amount of compensation depends upon the load and the length of the feeder, it is evident that in many cases good regulation is only possible when each feeder is independently regulated."

"In addition to the voltage drop in feeders and in primary mains there is a voltage drop in transformers and secondary mains. These voltage drops cannot be compensated for by regulators, but must be taken care of by

[1] From Circular of the Bureau of Standards, No. 56 on "Standards for Electric Service," pp. 16 and 17.

selection of transformers with good regulation; careful supervision of conditions of load on each transformer of the system, and the proper size of conductors for the secondary mains. Transformers sometimes become overloaded, due to rapid growth of individual customer's loads, the number of customers to be supplied from one distributing point, and insufficient station records of transformer installations and their connected loads. It should be noted, too, that the regulation of transformers is dependent on the power-factor of the load, and some transformers, while giving good regulation on non-inductive loads, may cause large voltage drop at full load on mixed motor and lighting load."

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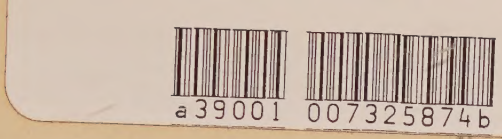
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